Chapter 11

Gases

Why study gases?

- To understand an important part of real world phenomena.
- A lot of industrial chemistry involves gases.
- They are the simplest of the three phases to understand and quantitatively model (i.e. predict) their behavior
- A good case study to see some scientific concepts and principles in action

A Gas

- Uniformly fills any container.
- Easily compressed.
- Mixes completely with any other gas.
- Exerts pressure on its surroundings.

Pressure

$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

SI units = Newton/meter^2 = 1 Pascal (symbol: Pa)

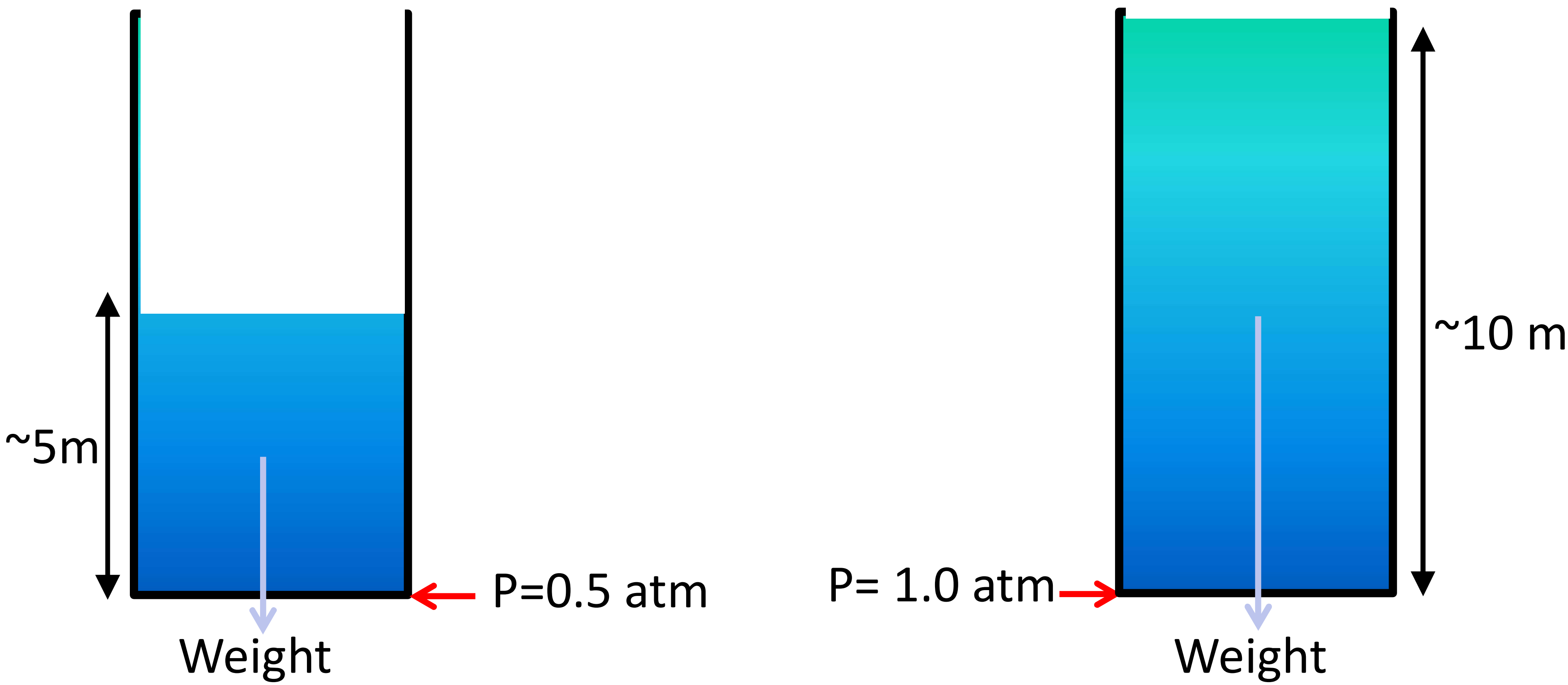
1 standard atmosphere (symbol: atm) = 101,325 Pa

1 standard atmosphere = 1 atm = 760 mm Hg = 760 torr

The “normal” atmospheric pressure is approximately equal to a “standard atmosphere” or simply “atmosphere”

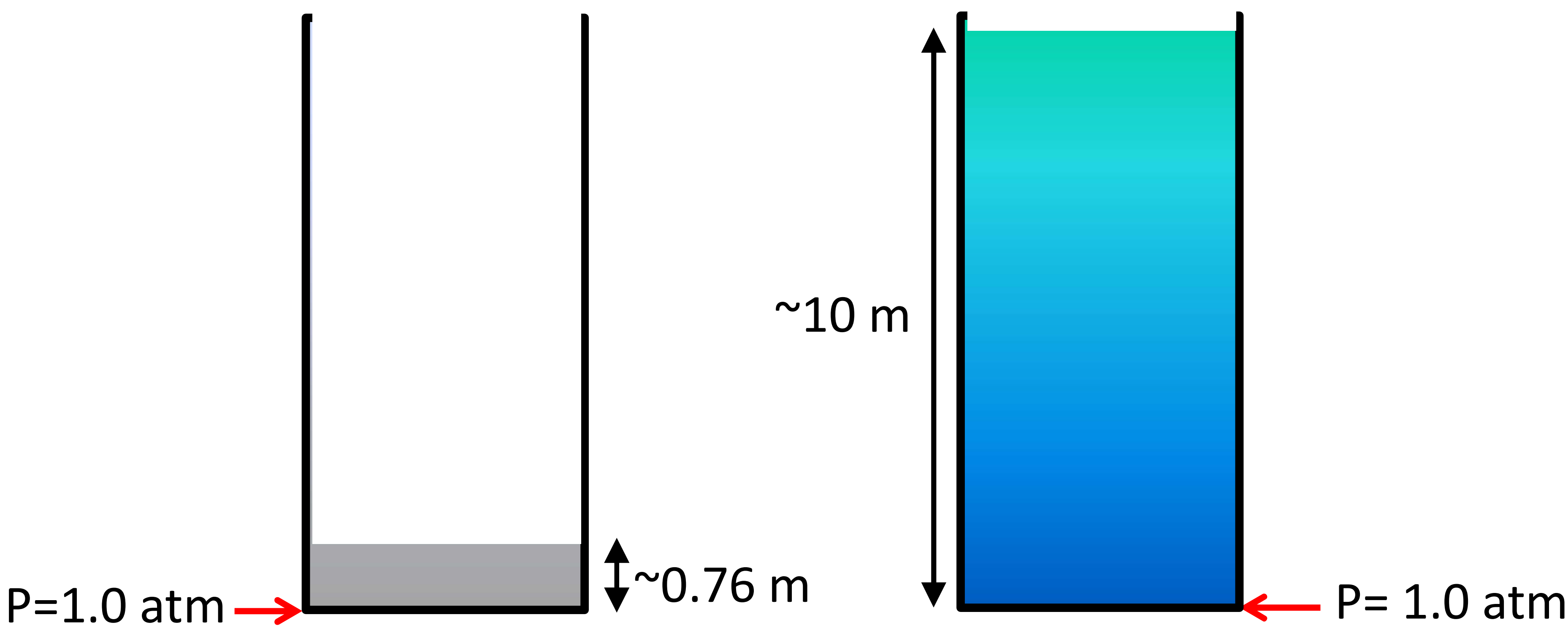
But first … let's understand how pressure is generated in dense materials (liquids and solids): by an external force

- The weight (due to gravitational force) of liquid "sitting" above an area generates hydrostatic pressure.
- It is literally the weight of the liquid column divided by the area on which it "sits"



Dense liquids require less height to generate the same pressure as a less dense liquid

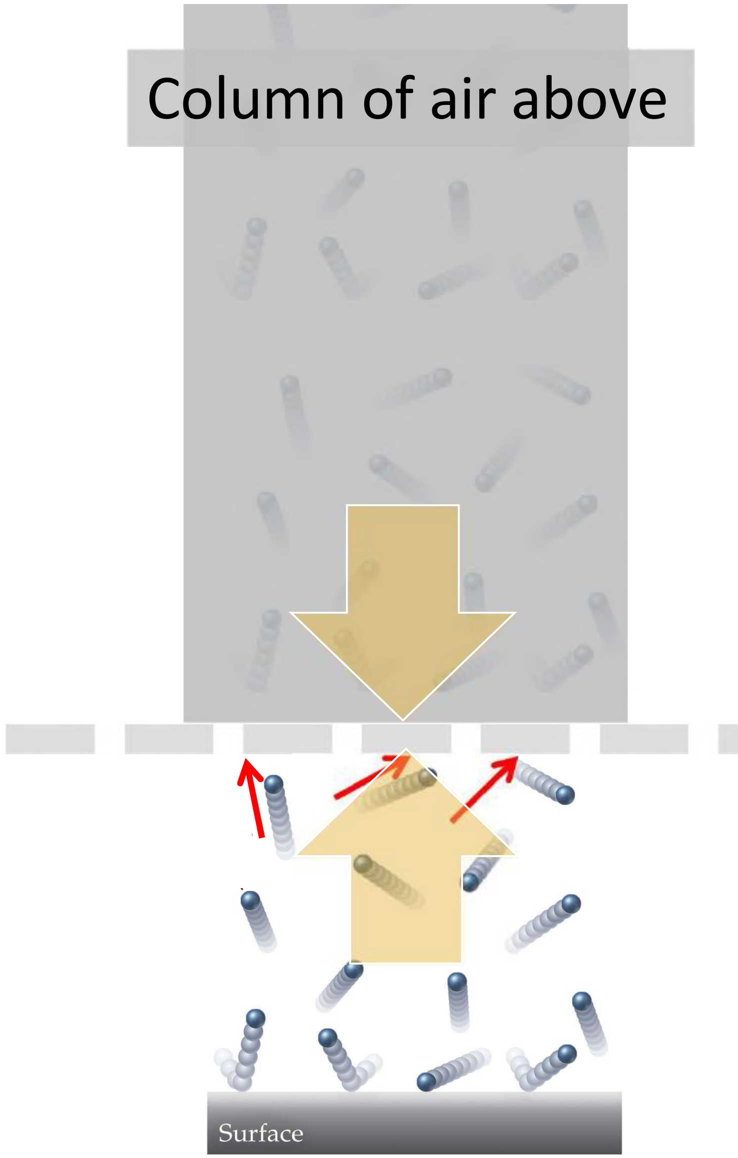
- Mercury is 13.6 denser than water
- 760 mm (0.760 m) generates the same hydrostatic pressure as ~10 m column of water



Normally, we can ignore the effect of gravity on the behavior of a human-scale sample of gas

But a planet-wide atmosphere is of course held in place and "compressed" to a pressure by gravity

Air around us is holding up the column of air above us in the atmosphere by applying a force in the opposite direction



So the air at the surface of the earth owes its pressure to the weight of the "column of air" sitting on top of any given point

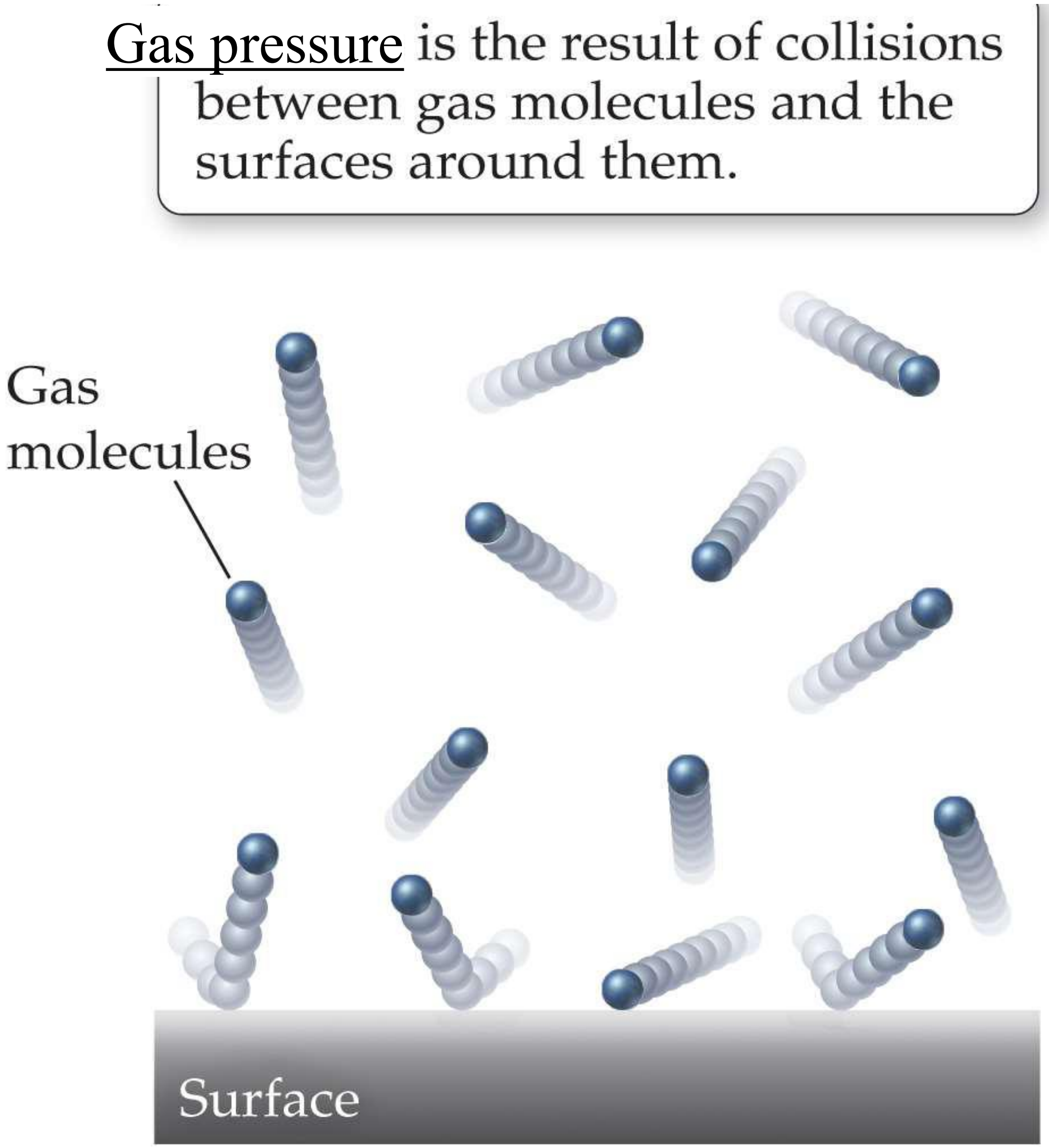
But …

- What if we take a sample of the air and put it in a sealed box without altering its pressure?
- Will it have little or no pressure because there is no "column of air" above it?

No, it will generate the same pressure

- It will retain its pressure because the pressure of a gas is actually due to its particles bouncing against the walls of its container.

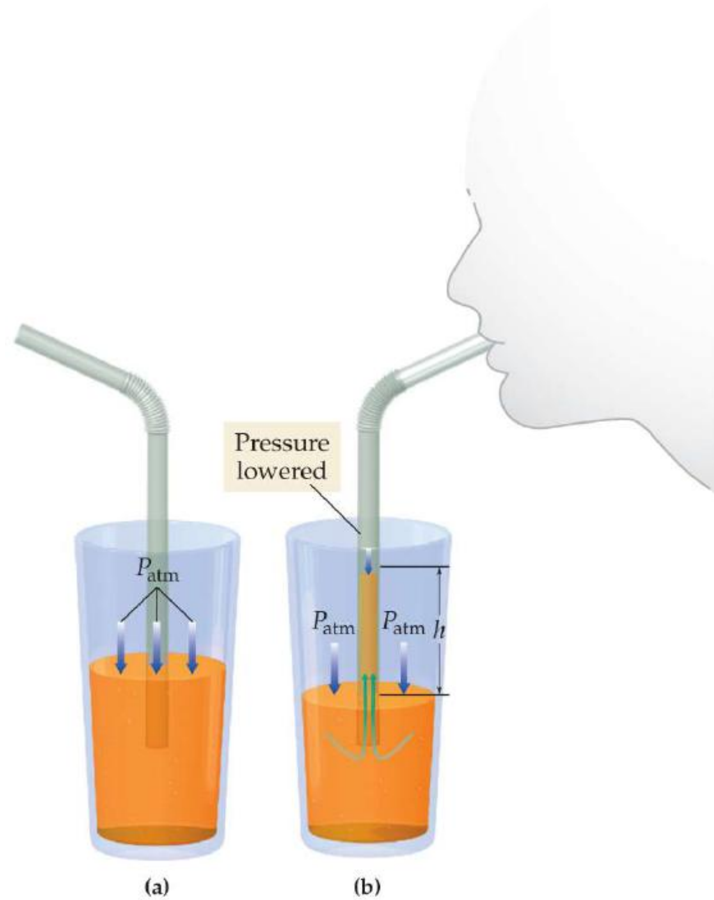
We will deal with human-scale samples where gravity is not important

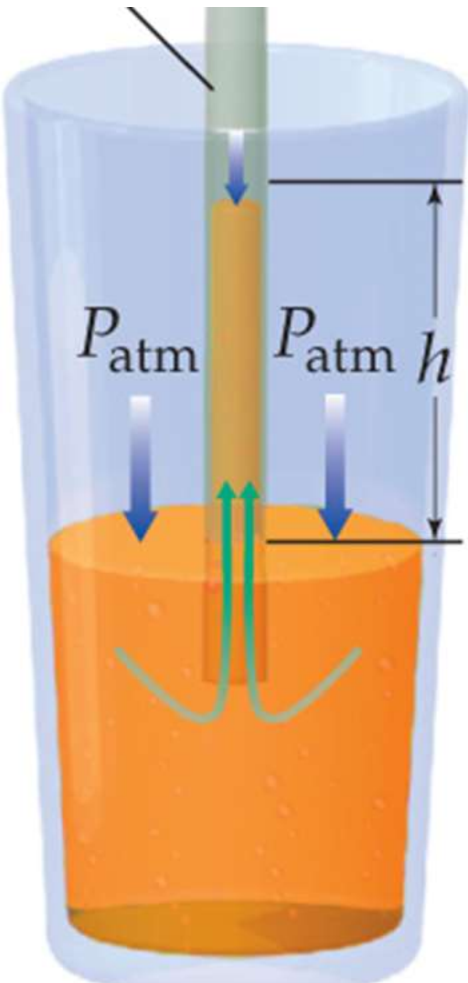


Heard of barometers? Let's first understand straws

(a) When a straw is put into a glass of orange soda, the pressure inside and outside the straw is the same, so the liquid levels inside and outside the straw are the same.

(b) When a person sucks on the straw, the pressure inside the straw is lowered. The greater pressure on the surface of the liquid outside the straw pushes the liquid up the straw.

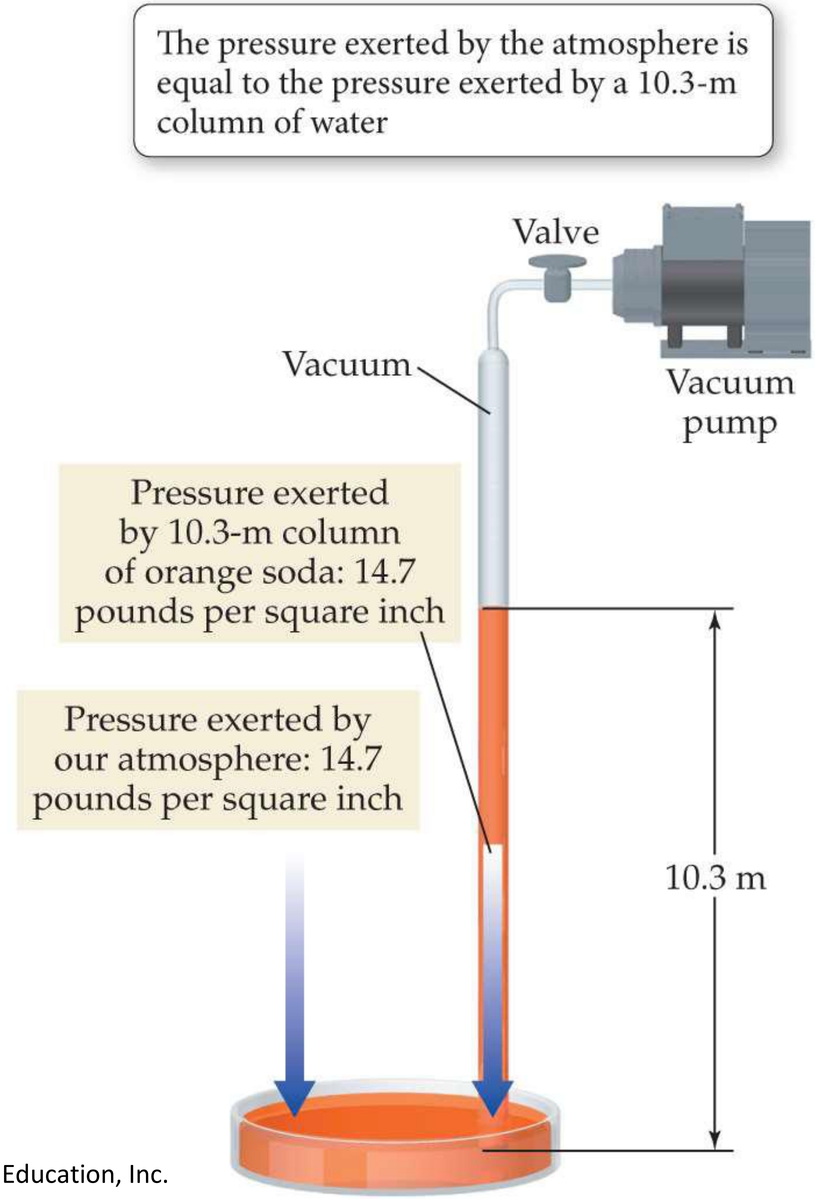




Note:

- Pressure is directionless
- If you apply it to a fluid, it is transmitted in all directions
- Atmosphere pressing "down" on the liquid surface translates to liquid pressing up into the straw

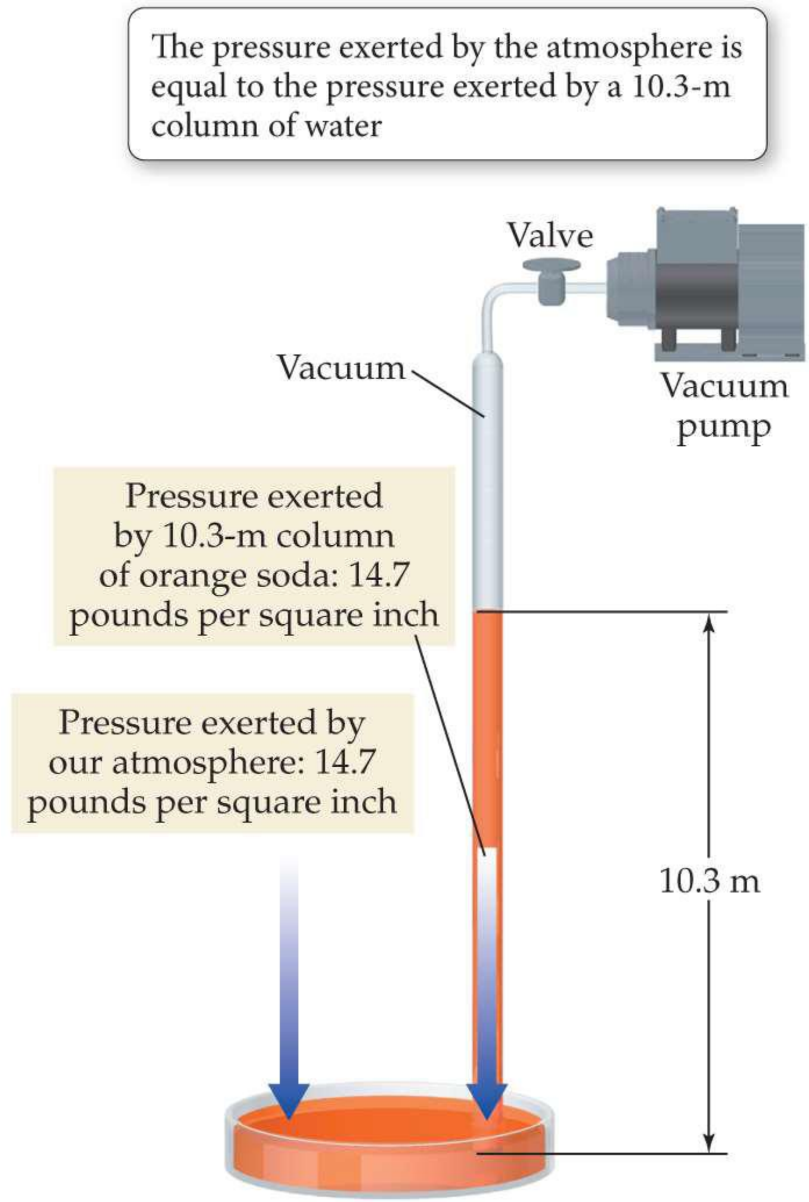
- Even if you formed a perfect vacuum with a pump, atmospheric pressure could only push orange soda to a total height of about 10 m.
- A column of water (or soda) 10.3 m high exerts the same pressure as the gas molecules in the atmosphere.



Heard of barometers? Let’s first understand straws

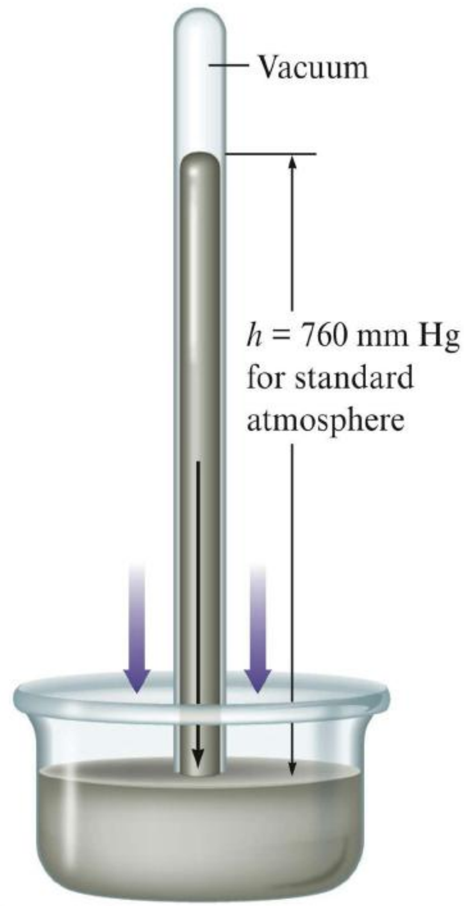
- We could measure the height of the water column to measure the atmospheric pressure
- If the atmospheric pressure is holding up 10.342 meters, we could even make it a unit and say “atmospheric pressure is 10.342 meters water”
- But it would be a huge, unwieldy instrument
- **A much denser liquid would have much shorter height:**

  Mercury!



Barometer

- Device used to measure atmospheric pressure.
- Mercury flows out of the tube until the pressure of the column of mercury standing on the surface of the mercury in the dish is *equal* to the pressure of the air on the rest of the surface of the mercury in the dish.



Weight of the mercury column is balanced by the force due to air pressure

The width of the column doesn't matter. The wider it is, the heavier the column, but also the larger the force due to air pressure (F=P·A)

Pressure Conversions: An Example

The pressure of a gas is measured as 2.5 atm. Represent this pressure in both torr and pascals.

$$(2.5\ \cancel{\text{atm}}) \times \left(\frac{760\ \text{torr}}{1\ \cancel{\text{atm}}}\right) = 1.9 \times 10^{3}\ \text{torr}$$

$$(2.5\ \cancel{\text{atm}}) \times \left(\frac{101{,}325\ \text{Pa}}{1\ \cancel{\text{atm}}}\right) = 2.5 \times 10^{5}\ \text{Pa}$$

The state of a gas is defined by:
Pressure, Volume, Temperature, and Moles

The **4 properties** (variables) that define the state of a gas are:

Pressure (**P**)

Volume (**V**)

Temperature (**T**)

Number of moles (**n**)

"Gas Laws" relate those 4 properties to one another

Liquid Nitrogen and a Balloon

- This is an observation (a fact).
- It does NOT explain "why," but it does tell us "what happened."
- Gas laws can be deduced from observations like these.

We saw the units of pressure. How about the units of other properties?

Temperature:

Always **Kelvins** (K)!

Using Celcius or Fahrehheit would be <u>disastrous</u>!

Volume:

Normally **liters** (L), but other volume units are alright, as long as you are “careful”

Number of Moles:

moles (mol)

Boyle's Law

For a given amount of gas, at a given temperature:

- **volume and pressure are inversely related**
- increasing its volume decreases its pressure

-- At constant temperature (T) and moles (n) of gas

$$P \times V = [\text{a constant value}]$$

If at first $P=P_1$ and $V=V_1$ $\Rightarrow$ $P_1 \times V_1 = [\text{a constant value}]$

and then $P=P_2$ and $V=V_2$ $\Rightarrow$ $P_2 \times V_2 = [\text{same constant value}]$

$\Rightarrow$ $$\boxed{P_1 \times V_1 = P_2 \times V_2}$$ Boyle's Law

Boyle's Law, $P_1V_1 = P_2V_2$, can be used when a there is a change in P or V at constant temperature and amount.

If we know three of the four quantities P_1, V_1,P_2, V_2, we can solve for the unknown forth one.

Example:

A sample of helium gas occupies 12.4 L at 23˚C and 0.956 atm. What volume will it occupy at 1.20 atm assuming that the temperature stays constant?

We recognize this as a "Boyle's Law" problem because the temperature (T) and amount of gas (n) is being held constant, and pressure (P) and volume (V) are changing

$$P_1 V_1 = P_2 V_2$$

0.956 atm 12.4 L 1.2 atm ?

$$(0.956)(12.4) = (1.2)\,V_2 \Longrightarrow V_2 = 9.88\text{ L}$$

Practice:
Check your solution on next page

- A cylinder with a movable piston has a volume of 7.25 L at 4.52 atm. What is the volume at 1.21 atm?

Practice: A cylinder with a movable piston has a volume of 7.25 L at 4.52 atm. What is the volume at 1.21 atm?

Given: $V_1 = 7.25$ L, $P_1 = 4.52$ atm, $P_2 = 1.21$ atm

Find: V_2, L

Conceptual Plan: $V_1, P_1, P_2 \rightarrow V_2$

$$V_2 = \frac{P_1 \; V_1}{P_2}$$

Relationships: $P_1 \cdot V_1 = P_2 \cdot V_2$

Solution:

$$V_2 = \frac{P_1 \; V_1}{P_2}$$

$$= \frac{(4.52\ \cancel{\text{atm}})\ (7.25\ \text{L})}{(1.21\ \cancel{\text{atm}})} = 27.1\text{L}$$

Check: because P and V are inversely proportional, when the pressure decreases ~4x, the volume should increase ~4x, and it does

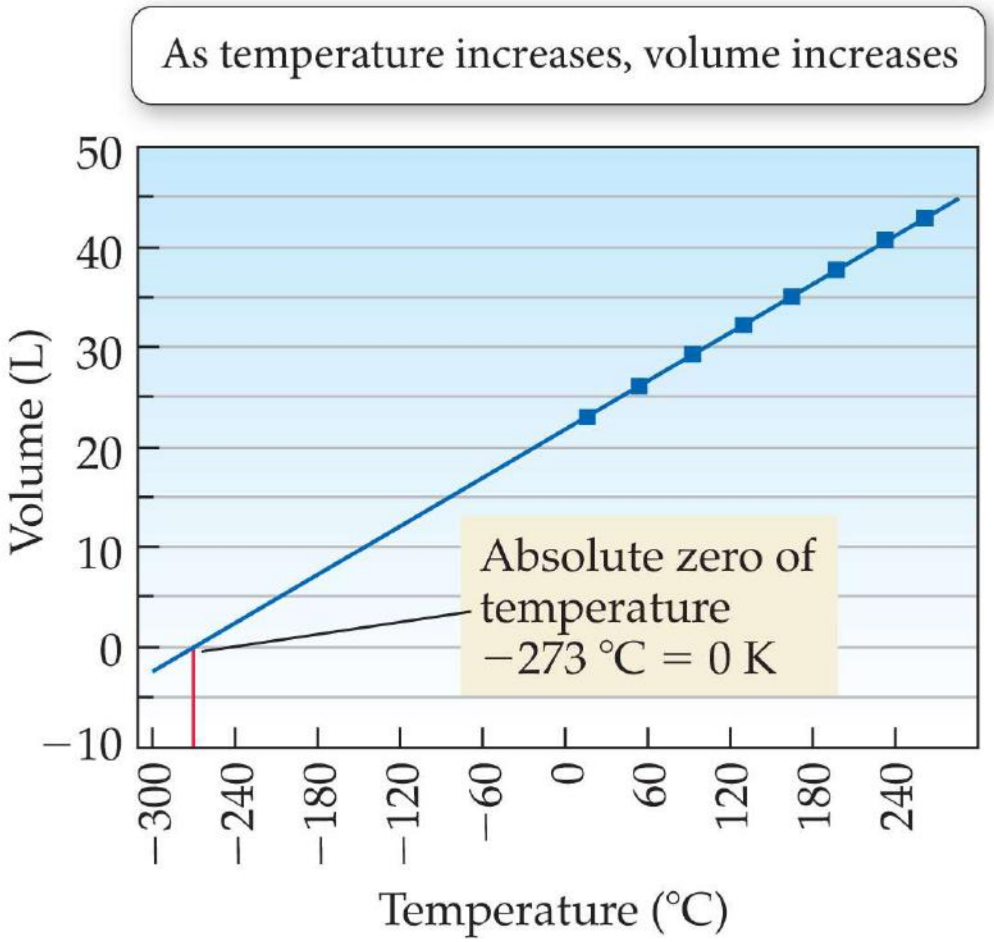
Liquid Nitrogen and a Balloon

A decrease in temperature was accompanied by a decrease in volume of the gas in the balloon.

Hot air less dense than cool air. That's why hot air rises

"Less dense" means larger volume for the same amount of gas.

Higher temperature goes with larger volume



V is linear function of T:

$$V = aT + b$$

At 0°C, volume is not zero (it's the y-intercept, *b*)

Extrapolating to -273.15 °C gives zero volume.

That is how the **Kelvin** scale was first conceived:

A temperature scale that makes V simply proportional to T

When T is zero **Kelvin**, V is zero.

Charles's Law

At constant P and moles (n) of gas, V is proportional to T:

$$V \propto T \quad \text{in Kelvin!}$$

meaning $\dfrac{V}{T}$ = [a constant value]

If at first $T = T_1$ and $V = V_1$ $\Rightarrow$ $\dfrac{V_1}{T_1}$ = [a constant value]

and then $T = T_2$ and $V = V_2$ $\Rightarrow$ $\dfrac{V_2}{T_2}$ = [same constant value]

$$\Rightarrow \quad \boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}} \quad \text{Charles's Law}$$

Charles's Law, $T_1 / V_1 = T_2 / V_2$, can be used when a there is a change in T or V at constant pressure and amount.

If we know three of the four quantities T_1, V_1,T_2, V_2, we can solve for the unknown forth one.

Example:

Suppose a balloon containing 1.30 L of air at 24.7˚C is placed into a beaker containing liquid nitrogen at 77.0 kelvins. What will the volume of the sample of air become (at constant pressure)?

We recognize this as a "Charles's Law" problem because the pressure (P) and amount of gas (n) is being held constant, and temperature (T) and volume (V) are changing

1.30 L

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

?

(24.7 + 273.15) K

77 K

$(1.30)\ (77.0)\ /\ (24.7 + 273.15) = V_2 \Rightarrow V_2 = 0.336\ \text{L}$

Practice:
Check your solution on next page

- A gas has a volume of 2.57 L at 0.00 ° C. What was the temperature at 2.80 L?

Practice: A gas has a volume of 2.57 L at 0.00°C. What was the temperature at 2.80 L?

Given: V_2 =2.57 L, T_2 = 0°C, V_1 = 2.80 L

Find: T_1

Conceptual Plan: $V_1, V_2, T_2 \rightarrow T_1$

$$T_1 = T_2 \frac{V_1}{V_2}$$

Relationships: $T(K) = t(°C) + 273.15,\quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$

Solution:

$$T_2 = 0.00 + 273.15$$

$$T_2 = 273.15\,K$$

$$T_1 = T_2 \frac{V_1}{V_2} = 273.15\,K \frac{2.57\,\cancel{L}}{2.80\,\cancel{L}} = 29\underline{7}.6\,K$$

$$t_1 = T_1 - 273.15$$

$$t_1 = 29\underline{7}.6 - 273.15$$

$$t_1 = 24\,°C$$

Check: because T and V are directly proportional, when the volume decreases, the temperature should decrease, and it does

Avogadro's 's Law

More gas particles (measured by moles, *n*) at a given pressure and temperature would occupy more volume.

If there are no particles, volume is zero.

$$V \propto n \Rightarrow \frac{V}{n} = \text{[a constant value]}$$

If at first $V = V_1$ and $n = n_1$ $\Rightarrow \frac{V_1}{n_1} = \text{[a constant value]}$

and then $V = V_2$ and $n = n_2$ $\Rightarrow \frac{V_2}{n_2} = \text{[a constant value]}$

$$\Rightarrow \boxed{\frac{V_1}{n_1} = \frac{V_2}{n_2}}$$

Avogadro's Law

Example:

If 2.45 mol of argon gas occupies a volume of 89.0 L, what volume will 2.10 mol of argon occupy under the same conditions of temperature and pressure?

We recognize this as an "Avogadro's Law" problem because the pressure (P) and temperature (T) is being held constant, and number of moles (n) and volume (V) are changing

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

89.0 L → V_1

2.45 mol → n_1

? → V_2

2.10 mol → n_2

(89.0) (2.10) / (2.45) = V_2 ⇨ V_2 = 0.849 L

Avogadro's Law, $V_1 / n_1 = V_2 / n_2$, can be used when a there is a change in V or n at constant pressure and temperature.

If we know three of the four quantities V_1, n_1,V_2, n_2, we can solve for the unknown forth one.

Practice:
Check your solution on next page

- 0.225 mol sample of He has a volume of 4.65 L. How many moles must be added to give 6.48 L?

Practice: A 0.225 mol sample of He has a volume of 4.65 L. How many moles must be added to give 6.48 L?

$$V_1, V_2, n_1 \longrightarrow n_2$$

$$n_1 \bullet \frac{V_2}{V_1} = n_2$$

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

$$\text{mol added} = n_2 - n_1,$$

$$n_2 = \frac{n_1 \bullet V_2}{V_1}$$

$$= \frac{(0.225\ \text{mol}) \bullet (6.48\ \cancel{L})}{(4.65\ \cancel{L})} = 0.314\ \text{mol}$$

$$\text{moles added} = 0.314 - 0.225$$

$$\text{moles added} = 0.089\ \text{mol}$$

These “named” laws are useful when there is a change involving two variables, while two other variables are constant.

Important:

They also can be used when comparing two different samples of gases under different conditions.

- instead of the same gas undergoing a change

Combined gas law

We can combine Boyle's law and Charles' Law into a "combined" gas law:

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

Combined Gas Law

satisfies both laws when we hold constant the appropriate pairs of variables

The combined gas law applies only when the *amount* of gas is constant.
The temperature must be expressed in kelvins.

- A sample of gas has an initial volume of 158 mL at a pressure of 735 mm Hg and a temperature of 34 °C.
- If the gas is compressed to a volume of 108 mL and heated to a temperature of 85 °C, what is its final pressure in millimeters of mercury?

- GIVEN:

P_1 = 735 mm Hg
t_1 = 34 °C
V_1 = 158 mL

t_2 = 85 °C
V_2 = 108 mL

FIND:
P_2 = ? mm Hg

- SOLUTION:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$P_2 = \frac{P_1V_1T_2}{T_1V_2}$$

$$T_1 = 34 + 273 = 307\ \text{K}$$

$$T_2 = 85 + 273 = 358\ \text{K}$$

$$P_2 = \frac{735\ \text{mm Hg} \times 158\ \cancel{\text{mL}} \times 358\ \cancel{\text{K}}}{307\ \cancel{\text{K}} \times 108\ \cancel{\text{mL}}}$$

$$= 1.25 \times 10^3\ \text{mm Hg}$$

We can combine Boyle's Law, Charles's Law, and Avogadro's Law to obtain:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} \Longrightarrow \frac{PV}{nT} = \text{constant}$$

Let's call it the "Gas Constant", and give it the symbol R

$$\Longrightarrow \frac{PV}{nT} = R \Longrightarrow \boxed{PV = nRT}$$

Ideal Gas Law

The units and value of R depends on the units of P, V, T, n
When the units are atm, L, K, mol, then

R = 0.08206 L atm/(mol K)

$$\boxed{PV = nRT}$$

Ideal Gas Law

It's called the "Ideal Gas Law" because it corresponds to an "idealized" version of actual gases:

- **Volume of the gas particles are negligible (zero)**
- **There are no attractive forces between gas particles**

The second approximation means:

- The particles are unaware of one another's existence
- Different gases mixed together are unaware of and undisturbed by one another

Other gas laws can be easily derived from PV=nRT

Red symbols can vary
Blue symbols are constant

$PV = nRT$ n , T constant

$PV = \text{constant}$ $P_1V_1 = P_2V_2$ Boyle's Law

$\frac{V}{T} = \frac{nR}{P}$ n , P constant

$\frac{V}{T} = \text{constant}$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ Charles' Law

$\frac{PV}{R} = nT$ P, V constant

$nT = \text{constant}$ $n_1T_1 = n_2T_2$ Un-named

Amontons' Law

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Can you derive it from the Ideal Gas Law?

How about a law relating P and n?

Gas laws and variables that are held constant

- There are only 4 variables to consider.
- and one equation (Ideal Gas Law) tying them together.

If we hold <u>3 variables constant</u>

The 4^{th} one is determined from the other three.

If we hold <u>2 variables constant</u>

The other 2 vary according to a “law” (they are either directly or inversely proportional) (Boyle’s Law, Charles’s Law, etc.)

If we hold <u>1 variable</u> constant

We get a “combined” gas law that ties together the other 3 variables

Deciding which law to use

Remember there are 4 variables: **P, V, T, n**

- If two are being held constant and two are changing, use the "named" laws like Boyle's Law, Charles Law, etc.
  - Sometimes you might need to derive an unnamed law from the ideal gas law

- If **P, V, T** are all changing, but **n** is constant, then use the "combined gas law".
  - You can also derive other "combined" gas laws if what's held constant is not **n**.

- If three variables are specified, and you are asked to find the fourth one, use the ideal gas law **PV = nRT**

Example:

An automobile tire at 23˚C with an internal volume of 25.0 L is filled with air to a total pressure of 3.18 atm. Determine the number of moles of air in the tire.

- We are given T, V, P
- No changes in them (three variables constant)

$\Rightarrow$ Use PV=nRT $\Rightarrow$ $n = \dfrac{PV}{RT}$

Make sure we use Kelvins for T: $T_1 = 23 + 273.15 = 29\underline{6}.15\ K$

$$n = \frac{\overset{P}{(3.18\ \text{atm})}\overset{V}{(25.0\ \text{L})}}{\underset{R}{(0.08206\ \text{L atm mol}^{-1}\ \text{K}^{-1})}\underset{T}{(29\underline{6}.15\ \text{K})}} = 3.27\ \text{mol}$$

Example:

What is the pressure in a 304.0 L tank that contains 5.670 kg of helium at 25˚C?

- We are given V, mass (which can give n), T
- No changes in them (Three variables constant)

⇨ Use PV=nRT ⇨ $P = \frac{nRT}{V}$

Make sure we use Kelvins for T: $T_1 = 25 + 273.15 = 29\underline{8}.15$ K

$$n = 5.670\ \text{kg} \ \frac{10^3\ \text{g}}{1\ \text{kg}} \ \frac{1\ \text{mol}}{4.0026\ \text{g}} = 141\underline{6}.4\ \text{mol}$$

$$P = \frac{\overset{n}{(1416.4\ \text{mol})}\overset{R}{(0.08206\ \text{L atm mol}^{-1}\ \text{K}^{-1})}\overset{T}{(29\underline{8}.15\ \text{K})}}{\underset{V}{(304.0\ \text{L})}} = 114\ \text{atm}$$

Example:

We have 121 mL of CO_2 gas at 27˚C and 1.05 atm. At what temperature (in ˚C) does it occupy a volume of 293 mL at a pressure of 1.40 atm?

- We are given V, T, P
- And another set of V, T, P
- Second T not known
- Three variables (V, T, P) change
- One variable (n) constant

⇨ Use the "combined gas law" $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Make sure we use Kelvins for T: $T_1 = 27 + 273.15 = 30\underline{0}.15$ K

We can keep mL for V as long as we use the same units for V_1 and V_2

$$\frac{(1.05)(121)}{(30\underline{0}.15)} = \frac{(1.40)(293)}{T_2} \Longrightarrow T_2 = 969\text{K} = (969\text{-}273)\ ˚\text{C}$$

$= \mathbf{696\ ˚C}$

Work in Kelvins, then convert to ˚C if required

Practice:
Check your solution on next page

- How many moles of gas are in a basketball with total pressure 24.3 psi, volume of 3.24 L at 25° C?

Practice: How many moles of gas are in a basketball with total pressure 24.3 psi, volume of 3.24 L at 25° C?

Given: V = 3.24 L, P = 24.3 psi, t = 25°C

Find: n (moles)

Conceptual Plan:

$$\text{P, V, T, R} \xrightarrow{n = \frac{PV}{RT}} \text{n}$$

Relationships:

1 atm = 14.7 psi

T(K) = t(°C) + 273.15

$$PV = nRT, \quad R = 0.08206 \frac{\text{atm L}}{\text{mol K}}$$

Solution:

$$P = 24.3\,\cancel{\text{psi}} \times \frac{1\,\text{atm}}{14.7\,\cancel{\text{psi}}} = 1.6\underline{5}31\,\text{atm}$$

$$T(K) = 25°C + 273.15 = 298\,K$$

$$n = \frac{PV}{RT} = \frac{(1.6\underline{5}31\,\cancel{\text{atm}})(3.24\,\cancel{\text{L}})}{\left(0.08206\,\frac{\cancel{\text{atm}}\,\cancel{\text{L}}}{\text{mol}\,\cancel{\text{K}}}\right)(298\,\cancel{\text{K}})} = 0.219\,\text{mol}$$

Check: 1 mole at STP occupies 22.4 L, because there is a much smaller volume than 22.4 L, we expect less than 1 mole of gas

Practice:
Check your solution on next page

Calculate the volume occupied by 637 g of SO_2 (MM 64.07 g/mol) at 6.08×10^4 mmHg and –23°C

Practice: Calculate the volume occupied by 637 g of SO_2 (MM 64.07) at 6.08 x 10^4 mmHg and –23°C

Given: m_{SO2} = 637 g, P = 6.08 x 10^4 mmHg, t = −23°C, *V*, L

Find: V (in Liters)

Conceptual Plan:

g → n : $\frac{1\,\text{mol}}{64.07\,\text{g}}$

P, n, T, R → V : $V = \frac{nRT}{P}$

Relationships: 1 atm = 760 mmHg, $PV = nRT$, $R = 0.08206\,\frac{\text{atm L}}{\text{mol K}}$

$T(\text{K}) = t(°\text{C}) + 273$, 1 mol SO_2 = 64.07 g

Solution:

$$P = 6.08 \times 10^4\,\cancel{\text{mmHg}} \times \frac{1\,\text{atm}}{760\,\cancel{\text{mmHg}}} = 80.0\,\text{atm}$$

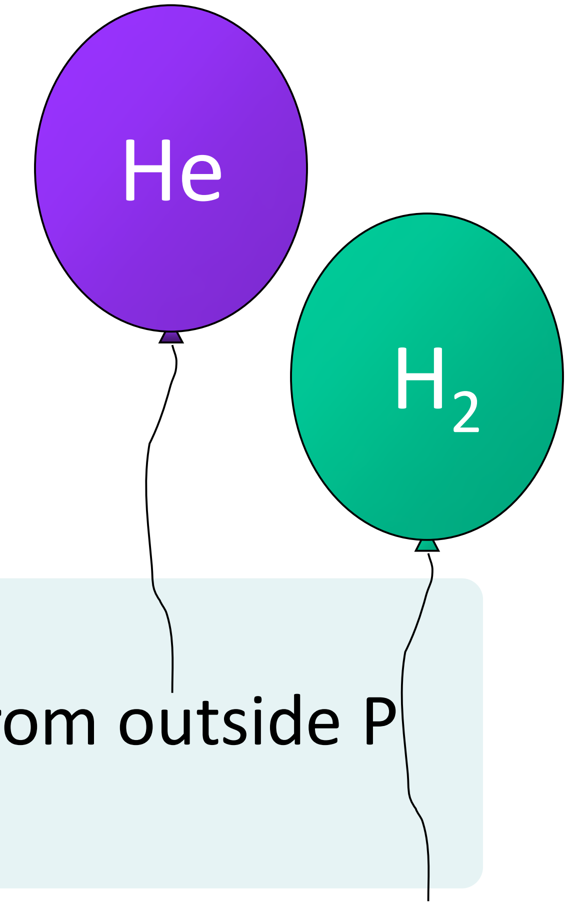
$$n = 637\,\cancel{\text{g}}\,SO_2 \times \frac{1\,\text{mol}\,SO_2}{64.07\,\cancel{\text{g}}} = 9.9\underline{4}2\,\text{mol}\,SO_2$$

$$T(\text{K}) = -23\,°\text{C} + 273 = 25\underline{0}.\,\text{K}$$

$$V = \frac{nRT}{P} = \frac{(9.9\underline{4}2\,\cancel{\text{mol}})(0.0821\,\frac{\cancel{\text{atm}}\,\text{L}}{\cancel{\text{mol}}\cdot\cancel{\text{K}}})(25\underline{0}.\,\cancel{\text{K}})}{(80.0\,\cancel{\text{atm}})} = 2.55\,\text{L}$$

Concept Practice:

You are holding two balloons of the **same volume**. One contains helium, and one contains hydrogen. Answer the upcoming questions as "**different**" or "**the same**" and explain.



A balloon is not a rigid container
It cannot maintain a pressure different from outside P

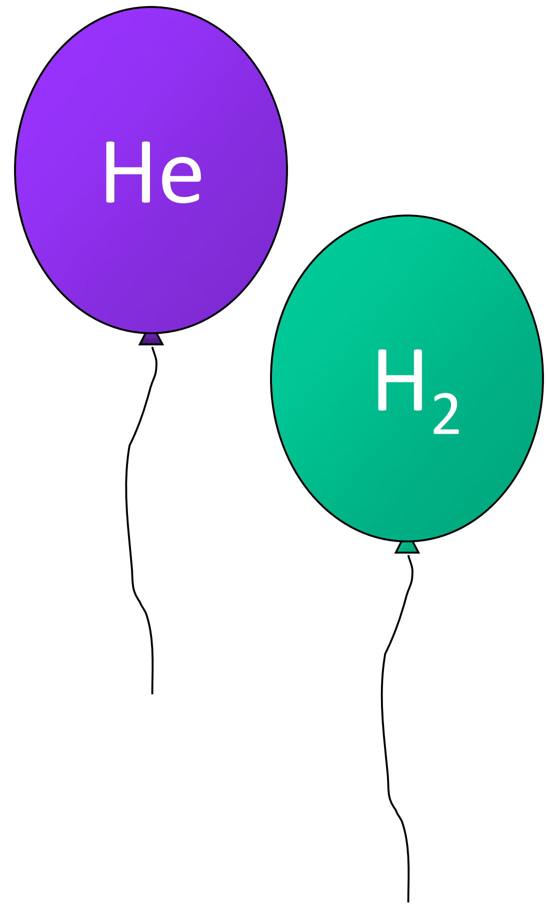
$$P_{balloon\ 1} = P_{room} = P_{balloon\ 2}$$

A balloon is not an insulating container
It cannot maintain a temperature different from outside T

$$T_{balloon\ 1} = T_{room} = T_{balloon\ 2}$$

Concept Practice:

The pressures of the gas in the two balloons are __the same___.



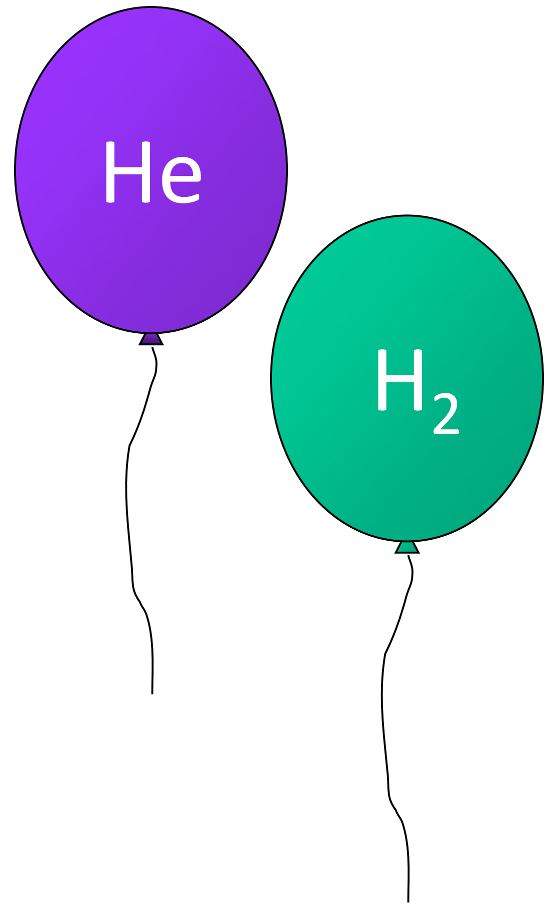
A balloon is not a rigid container

It cannot maintain a pressure that is different from outside pressure

$$P_{\text{balloon 1}} = P_{\text{room}} = P_{\text{balloon 2}}$$

Concept Practice:

The temperatures of the gas in the two balloons are __the same___.



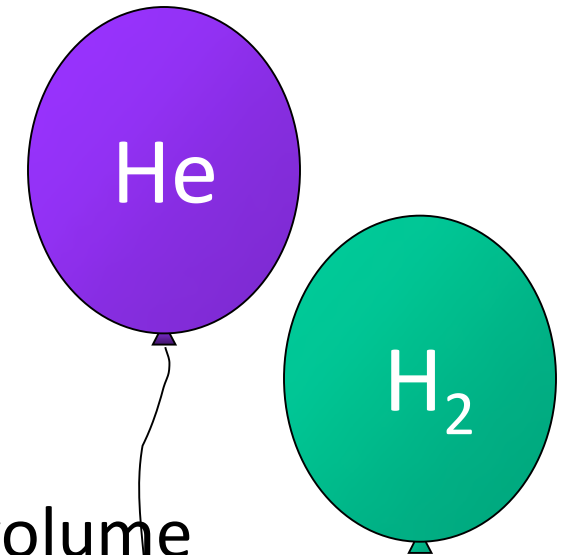
A balloon is not an insulating container

It cannot maintain a temperature that is different from outside temperature

$$T_{balloon\ 1} = T_{room} = T_{balloon\ 2}$$

Concept Practice:

The numbers of moles of the gas in the two balloons are **the same**.



- The question said they had the same volume

$$V_{\text{balloon 1}} = V_{\text{balloon 2}}$$

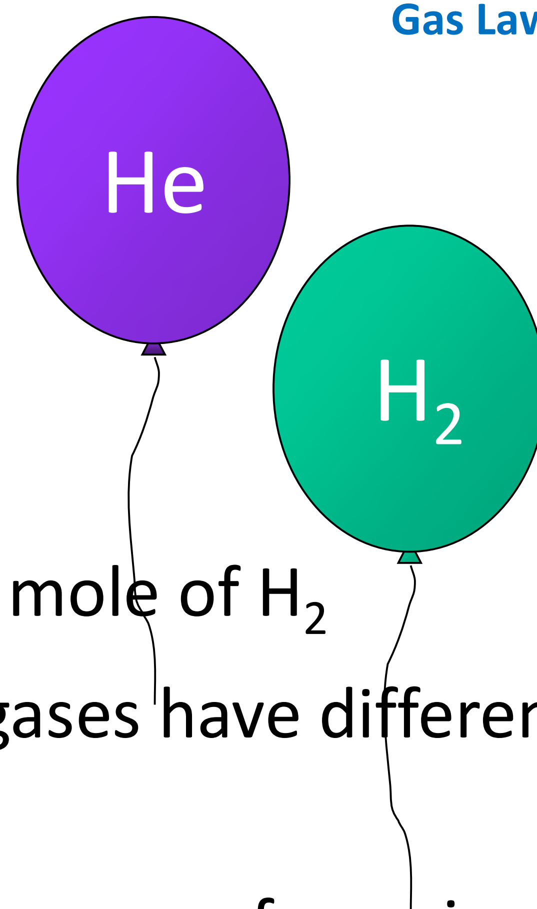
- We deduced that they have the same P and T values
- If 3 of the 4 variables are fixed, the 4^{th} one is also constant.
  - ➢ Because it is determined from the other 3 by PV = nRT

$$n = \frac{PV}{RT}$$

A given set of P, V, T values dictates a unique value for n

Concept Practice:

The densities of the gas in the two balloons are **different**___.



- A mole of He has a different mass than a mole of H_2
- Same number of moles of two different gases have different masses
- The one with greater molar mass has more mass for a given number of moles ($\boldsymbol{n}_{He} = \boldsymbol{n}_{H2}$) mass = (moles) × (molar mass)

  Since m.m.(He) = 4.00 g/mol; m.m. (H_2) = 2.02 g/mol
- Helium has almost double the mass of H_2 for a given $\boldsymbol{n}$
- Remember: density = **mass**/volume

Molar Volume of an Ideal Gas

By convention, a “standard temperature and pressure” (**STP**) has been defined as:

0˚C and **1 atm**

At **STP**, 1 mole of an ideal gas has a volume of **22.42 L**

- It is a convenient fact to memorize
- But we can readily find it by using the ideal gas law:

$$V = \frac{nRT}{P} = \frac{(1.000\ \cancel{\text{mol}})(0.08206\ \text{L}\cdot\cancel{\text{atm}}/\cancel{\text{K}}\cdot\cancel{\text{mol}})(273.2\ \cancel{\text{K}})}{1.000\ \cancel{\text{atm}}} = 22.42\ \text{L}$$

- Knowing that 1 mole of gas has 22.42 L of volume at 273.15 K and 1 atm gives us the same information as knowing the gas constant *R.*
- In fact, to measure *R* experimentally, one would measure the V for a given n, P, T.
- STP is simply a recognized point out of an infinite set of possible n, P, T, V combinations, constrained only by the value of *R*.

$$PV = nRT$$

$$R = \frac{PV}{nT} = \frac{(1\ \text{atm})\ (22.42\ \text{L})}{(1\ \text{mol})(273.15\ \text{K})} = 0.08208\,\frac{\text{atm L}}{\text{mol K}}$$

Last sig. fig. (uncertain) a bit different from the standard value (it’s normal)

Example:

A sample of oxygen gas has a volume of 2.50 L at STP. How many grams of O_2 are present?

Before we find the mass, we need to find the moles

We can use the ideal gas law $PV=nRT$

$$n = \frac{PV}{RT} = \frac{(1\ \text{atm})\ (2.50\ \text{L})}{(0.08206)(273.15\ \text{K})} = 0.11\underline{1}5\ \text{mol}$$

But it's a lot simpler to use the molar volume at STP:

$$2.50\ \text{L} \times \frac{1\ \text{mol}}{22.42\ \text{L}} = 0.11\underline{1}5\ \text{mol}$$

m.m. O_2 = 32.00 g/mol

$$0.11\underline{1}5\ \text{mol} \times \frac{32.00\ \text{g}}{1\ \text{mol}} = \boxed{3.57\ \text{g}}$$

Example:

What is the density of F_2 at STP (in g/L)?

At STP, for 1 mol of gas, **V=22.42 L**

Since density is mass/volume, we convert mol to mass:

m.m. F_2 = 38.00 g/mol

$$\textbf{mass} = 1\ \text{mol} \times \frac{38.00\ \text{g}}{1\ \text{mol}} = \mathbf{38.00\ g}$$

$$d = \frac{\text{mass}}{V} = \frac{38.00\ \text{g}}{22.42\ \text{L}} = 1.695\ \text{g/mol}$$

Practice:

Calculate the density of $N_2(g)$ at STP

Check your solution on next page

Practice:

Calculate the density of $N_2(g)$ at STP

We know the molar volume (volume per mol) of an ideal gas:

22.42 L/mol

which means we know the no. of moles per volume:

1 mol/22.42 L

and when we know the molar mass of the gas:

28.01 g/mol

we can convert moles-per-liter to grams-per-liter

$$\frac{1\ \cancel{\text{mol}}}{22.42\ \text{L}} \times \frac{28.01\ \text{g}}{1\ \cancel{\text{mol}}} = 1.249\ \text{g/L}$$

Gas Density

$$\text{mass} \times \frac{1\,\text{mol}}{\text{molar mass}} = \text{moles}$$

$$\text{moles} = \frac{\text{mass}}{\text{molar mass}}$$

$$\text{density} = \frac{\text{mass}}{V}$$

$$P \times V = n \times R \times T$$

$$P \times V = \frac{\text{mass}}{\text{molar mass}} \times R \times T$$

Rearrange to obtain mass/V:

$$\frac{\text{mass}}{V} = \frac{P \times (\text{molar mass})}{R \times T}$$

$$\text{density} = \frac{P \times (\text{molar mass})}{R \times T}$$

Density is directly proportional to molar mass

Partial Pressures

Partial Pressure of an ideal gas:

- The pressure it would exert if it were alone

For a mixture of ideal gases in a container,

$$P_{Total} = P_1 + P_2 + P_3 + \ldots$$

The total pressure exerted is the sum of the pressures that each gas would exert if it were alone.

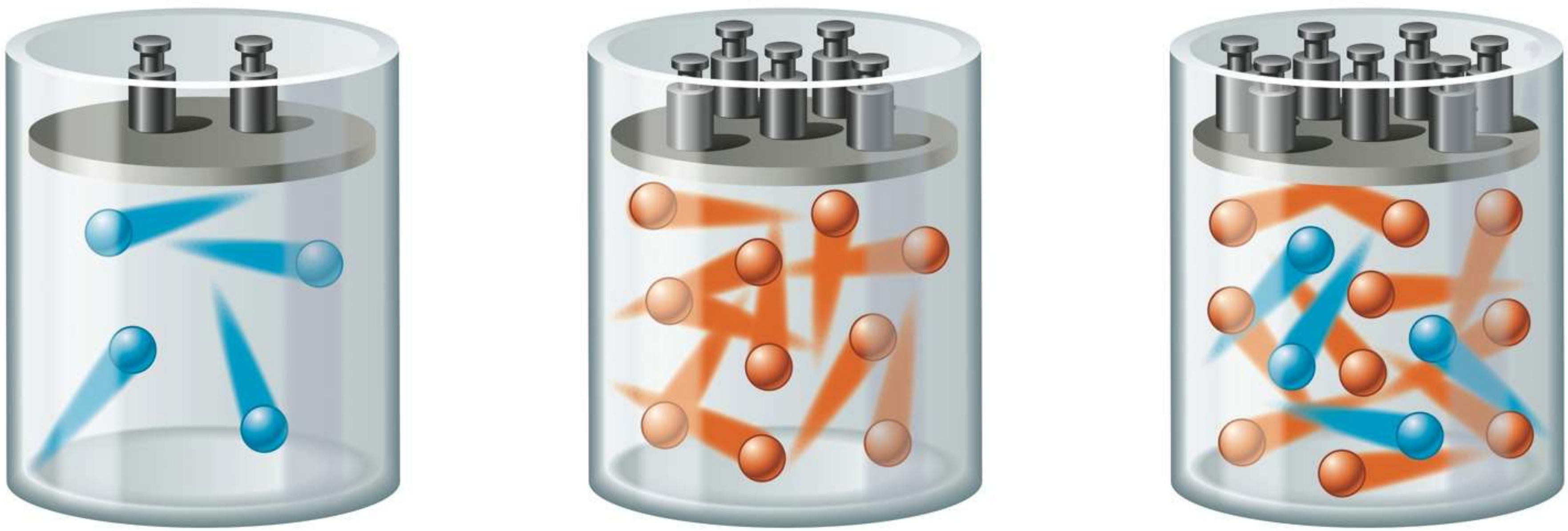
- Remember that ideal gas particles don't interact, so they are "unaware" of other gases in the same container
- So the presence of another gas has no effect on what pressure a gas would generate by itself. The pressures simply add up.

Partial Pressures

Gas 1 alone — P_1

Gas 2 alone — P_2

Gas 1 + Gas 2 — $P = P_1 + P_2$



Example:

27.4 L of oxygen gas at 25.0˚C and 1.30 atm, and 8.50 L of helium gas at 25.0˚C and 2.00 atm were pumped into a tank with a volume of 5.81 L at 25˚C. Calculate the partial pressures of oxygen and helium in the tank, as well as the total pressure.

Both gases are being transferred from their original container to the same new tank (new V). We will think about them separately.

Each gas preserves its number of moles, and temperature remains the same at 25.0˚C. This means constant **n** and **T**.

At constant n and T, P and V are allowed to change, following Boyle's Law: $P_1 V_1 = P_2 V_2$

For O_2: $(1.30)(27.4) = P_2 (5.81)$ $P_2 = 6.13 \text{ atm} = \mathbf{P_{O2}}$

For He: $(2.00)(8.50) = P_2 (5.81)$ $P_2 = 2.93 \text{ atm} = \mathbf{P_{He}}$

$\mathbf{P_{total}} = P_{O2} + P_{He} = 6.13+2.93 = 9.06 \text{ atm}$

Mole Fractions

mole fraction of A: $\boxed{x_A = \dfrac{n_A}{n_{total}}}$

Mole fraction is particularly useful with partial pressures

$$P_A = x_A\, P_{total}$$

$$\boxed{x_A = \dfrac{P_A}{P_{total}}}$$

Mole fractions are also like "pressure fractions"

$$x_A + x_B + x_C + \cdots = 1$$

$$P_A + P_B + P_C + \cdots = P_{total}$$

Example

The partial pressures of CH_4, N_2, and O_2 in a sample of gas were found to be 135 mmHg, 508 mmHg, and 571 mmHg, respectively. Calculate the mole fraction of nitrogen.

Mole fractions are also like “pressure fractions”

$$x_{N_2} = \frac{P_{N_2}}{P_{total}}$$

$$P_{total} = P_{CH_4} + P_{N_2} + P_{O_2} = 135+508+571 = 1214 \text{ mmHg}$$

$$x_{N_2} = \frac{508 \text{ mmHg}}{1214 \text{ mmHg}} = 0.418$$

Practice:
Check your solution on next page

- Find the partial pressure of neon in a mixture with total pressure 3.9 atm, volume 8.7 L, temperature 598 K, and 0.17 moles Xe.

Practice: Find the partial pressure of neon in a mixture with total pressure 3.9 atm, volume 8.7 L, temperature 598 K, and 0.17 mol Xe

Given: P_{tot} = 3.9 atm, V = 8.7 L, T = 598 K, Xe = 0.17 mol

Find: P_{Ne}, atm

Plan: n_{Xe}, V, T, R → P_{Xe} P_{tot}, P_{Xe} → P_{Ne}

$$P_{Xe} = \frac{n_{Xe} \quad R \quad T}{V}$$

$$P_{Ne} = P_{total} - P_{Xe}$$

Find P_{Xe} by applying the ideal gas law to Xe, connecting n_{Xe} to P_{Xe}

$$P_{Xe} = \frac{n_{Xe} \quad R \quad T}{V} = \frac{(0.17\ \cancel{mol})\left(0.08206\ \frac{atm \cdot \cancel{L}}{\cancel{mol \cdot K}}\right)(598\ \cancel{K})}{8.7\ \cancel{L}} = 0.9\underline{5}89\ \text{atm}$$

Then use the fact that partial pressures add up to the total pressure to find P_{Ne}

$$P_{Ne} = P_{total} - P_{Xe}$$
$$= 3.9\ \text{atm} - 0.9\underline{5}89\ \text{atm}$$
$$= 2.9\ \text{atm}$$

Practice:
Check your solution on next page

Find the mole fraction of neon in a mixture with total pressure 3.9 atm, volume 8.7 L, temperature 598 K, where we know that the partial pressure of Xe is 2.9 atm.

Practice: Find the mole fraction of neon in a mixture with total pressure 3.9 atm, volume 8.7 L, temperature 598 K, where we know that the partial pressure of Xe is 2.9 atm.

Given: P_{tot} = 3.9 atm, V = 8.7 L, T = 598 K, P_{Xe} = 2.9 atm

Find: x_{Ne} (unitless)

Plan: $P_{Xe}, P_{total} \rightarrow P_{Ne}$ $\qquad$ $P_{Ne}, P_{total} \rightarrow x_{Ne}$

$$P_{Ne} = P_{total} - P_{Xe}$$

$$x_{Ne} = \frac{P_{Ne}}{P_{tot}}$$

Volume and temperature information is not needed!

$$P_{Ne} = P_{total} - P_{Xe} = 3.9 - 2.9 = 1.0 \text{ atm}$$

$$x_{Ne} = \frac{P_{Ne}}{P_{total}} = \frac{1.0 \cancel{\text{atm}}}{3.9 \cancel{\text{atm}}} = 0.26$$