Chapter 2

Measurement and Problem Solving

Significant Figures and Uncertainty

“Someone weighs 215 pounds”

Why wouldn’t it say 215.34706 pounds?

- The balance’s precision is limited to the nearest pound
  - The instrument itself may have fundamental limitations
  - It may be designed in a way that makes the person’s movements change the readings

- If the measurement can’t reasonably capture digits beyond whole pounds, those digits should not be reported

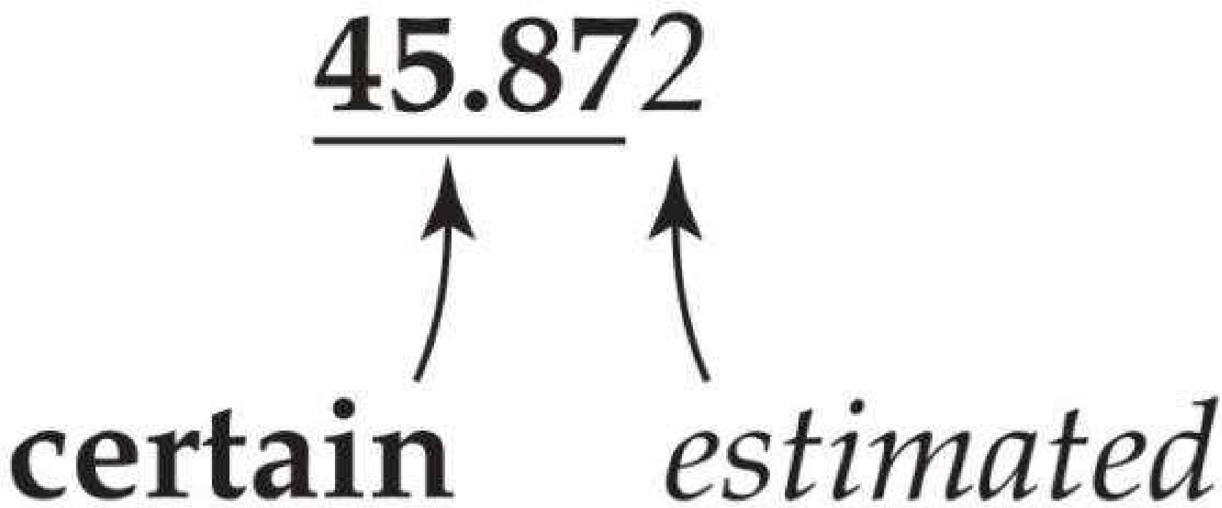
"Someone weighs 215 pounds"

Does it mean the reported 215 pounds is absolutely correct to the nearest pound? No.

- The last digit is still uncertain.
- If we are lucky, it is off only by 1, making the uncertainty ± 1
- But the uncertainty in the last digit may be any single-digit number, up to ± 9

The manual that comes with the balance might state its precision. For example ± 1 pounds

Reporting Scientific Numbers



- The last significant (i.e. reported) digit is estimated.
- The last digit still contains information; it just isn't perfect
- Any further digits are regarded as "garbage", and should not be reported

Estimating the last digit? How?

- An electronic instrument will estimate the last measurable digit automatically, but …
- If a human is making the measurement, we read down to the finest marks available, and then …
- We go one step further & literally “read between the lines”
  - We mentally divide the space between the finest marks into 10
  - Then we estimate the reading from these imaginary lines
- Why not 5 lines?
  - Because we have a decimal system

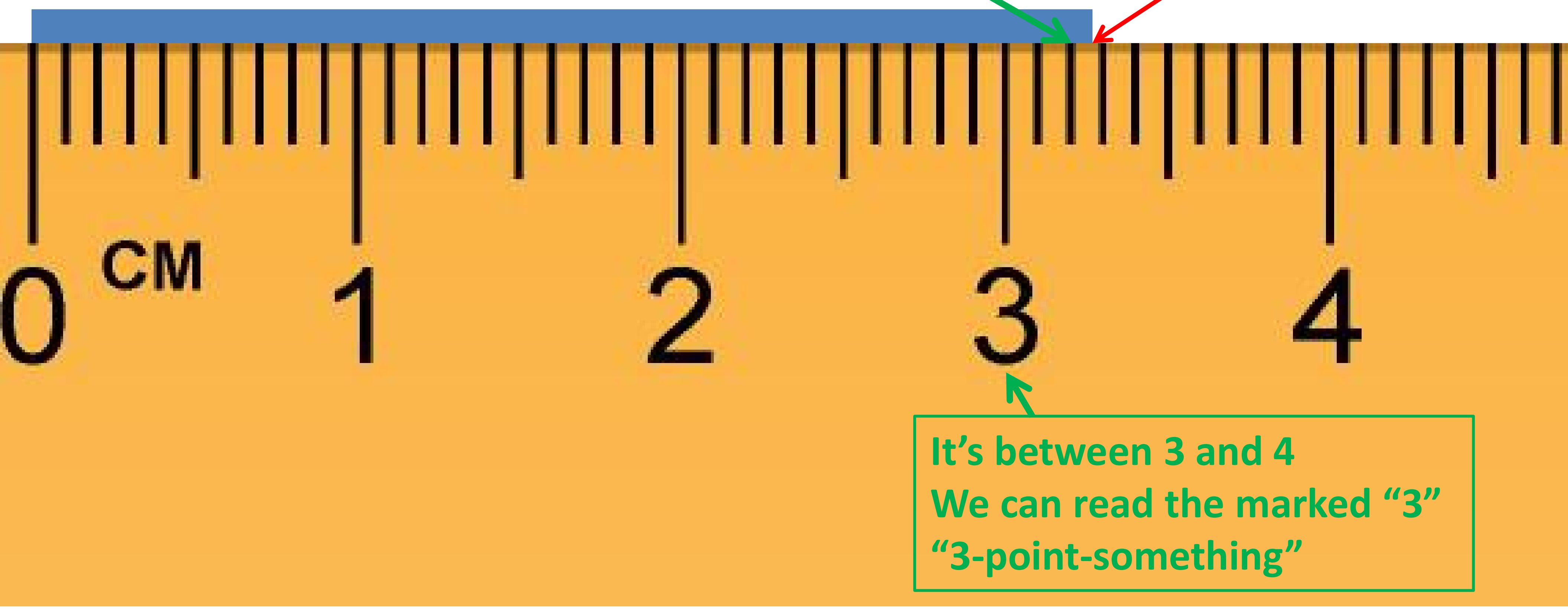
Anatomy of a measurement

3.27 cm

Significant but uncertain

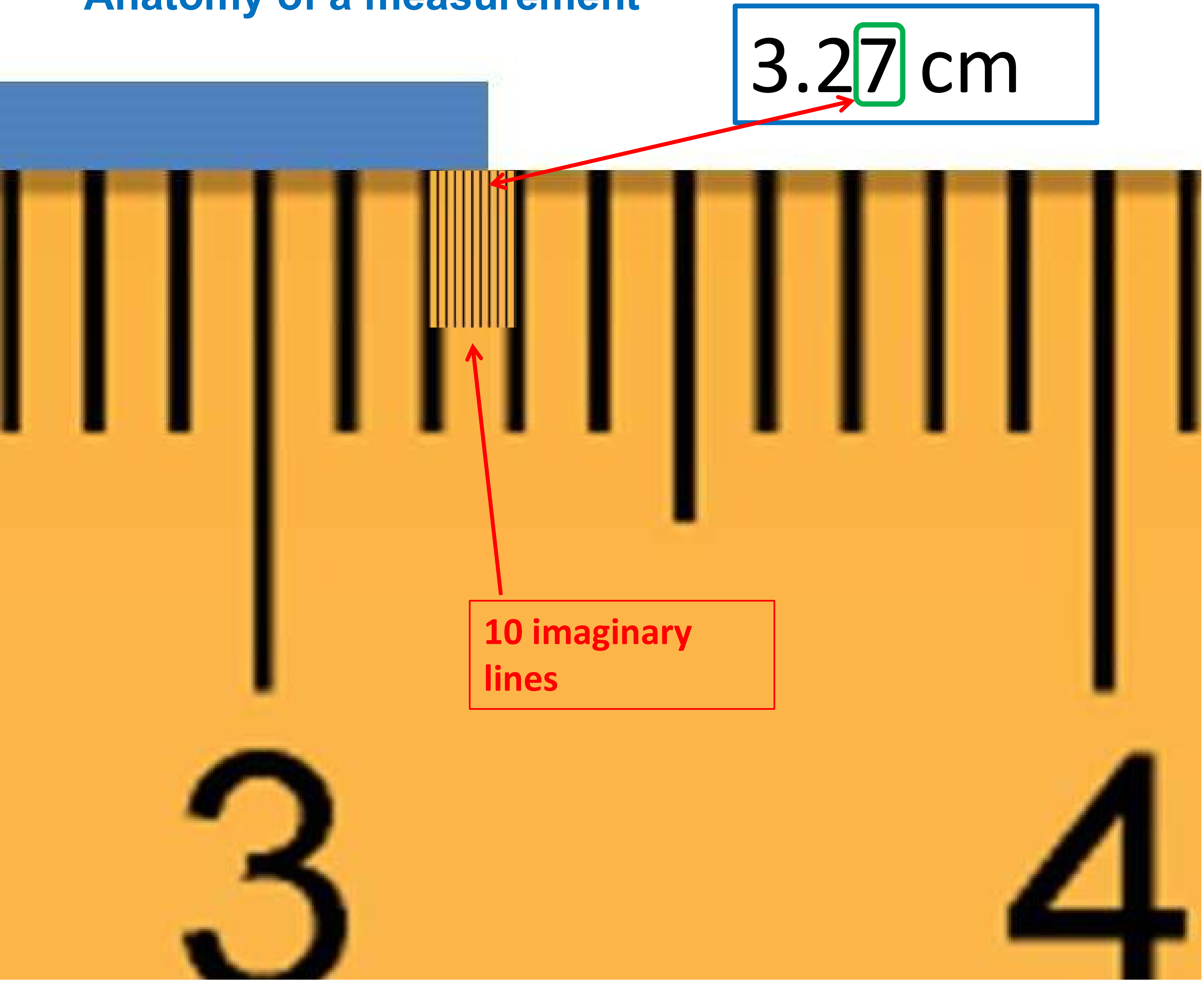
- It's between 3.2 and 3.3
- We can read the 2nd and 3rd marks
- "3-point-2-something"

- It's between 3.2 and 3.3
- We read between the 2nd and 3rd marks & estimate:
- "3-point-2-seven"
- or is it "3-point-2-eight"?
- Last, estimated digit is uncertain



It's between 3 and 4
We can read the marked "3"
"3-point-something"

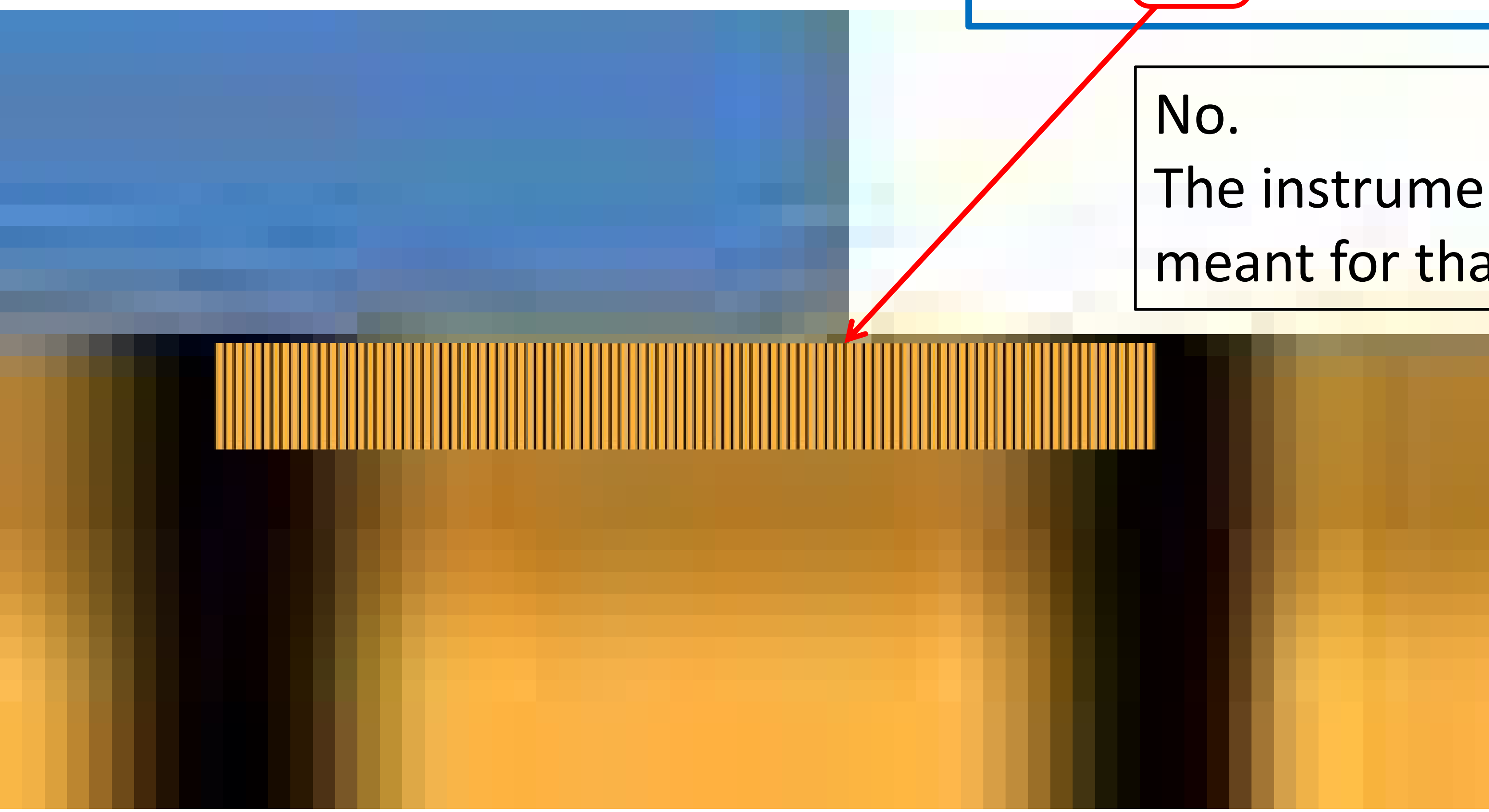
Anatomy of a measurement



But I zoomed in with my phone. Can I divide it into 100 lines then?

- The instrument is most likely not precise enough to justify it
- If it were, another level of finer marks would have been there

3.267 cm?



No.
The instrument is not meant for that precision

A weight measurement to tenth of a gram

- This balance has markings every 1 g.
- So we can estimate to the tenths place (i.e. 0.1 g)
- To estimate between markings, mentally divide the space into 10 equal spaces and estimate the last digit.

Last, estimated digit is significant but uncertain

1.2 grams

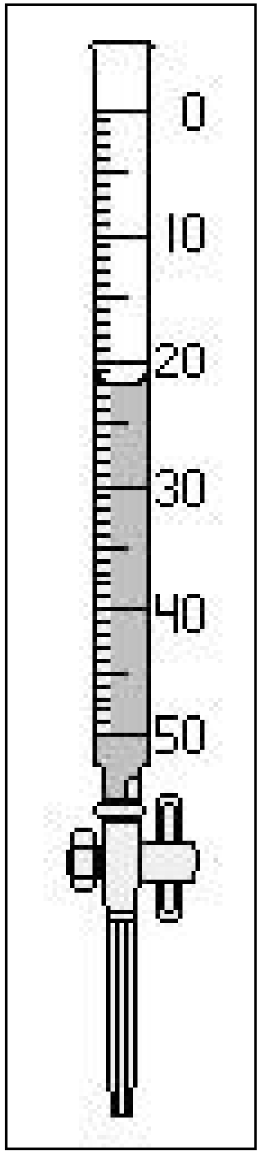
Seems reasonable

Somebody else could have read it as 1.3 g

Balance has marks every one gram

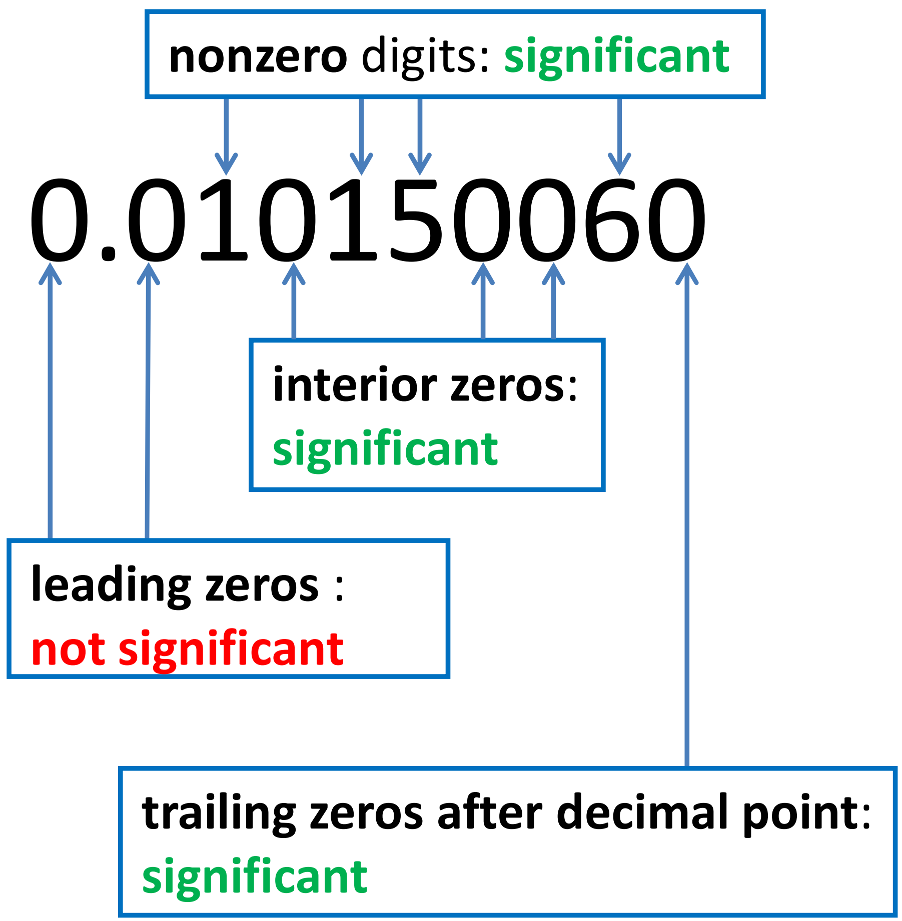
The buret is filled to the zero mark (at the top) with a solution and the solution is transferred to a beaker. What volume of transferred solution should be reported?

a) 20 mL
b) 22 mL
c) **22.0 mL**
d) 22.00 mL
e) 25 mL

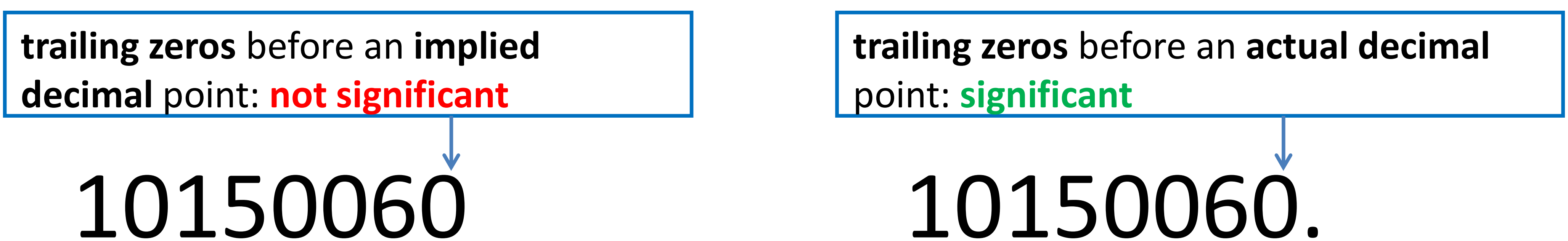


- Finest marks are placed 1 mL apart
- We can directly read the "ones"
- We can estimate the "tenths" (0.1)
- Even if the level is right at 22 mL, we must report the "tenths" digit as "0"

Significant Figures (“sig figs”) in a Measurement



- All **nonzero** digits **are significant.**
- **Interior zeros** (zeros between two nonzero numbers **are significant.**
  - Regardless of where the decimal point is
- **Trailing zeros** (zeros to the right of a nonzero number) that fall after a decimal point **are significant.**
- **Leading zeros** (zeros to the left of the first nonzero number) **are NOT significant.** They serve only to locate the decimal point.
- **Trailing zeros at the end** of a number, but before an implied (but not written) decimal point, should be assumed not-significant.
  - If a trailing zero before the last zero is significant, there is no way to indicate it!



Example : Determining the Number of Significant Figures in a Number

How many significant figures are in each of the following?

0.04450 m   4 sig. figs.; the digits 4 and 5, and the trailing 0

5.0003 km   5 sig. figs.; the digits 5 and 3, and the interior 0's

0.00002 mm   1 sig. figs.; the digit 2, not the leading 0's

10,000 m   Ambiguous, generally assume 1 sig. fig.

Practice – Determine the number of significant figures, the expected range of precision, and indicate the last significant figure (Check your answer on the next slide)

- 0.00120
- 120.
- 12.00

Tro: Chemistry: A Molecular Approach, 2/e

Practice – determine the number of significant figures, the expected range of precision, and indicate the last significant figure

- 0.0012*0* 3 sig. figs. 0.00119 to 0.00121
- 12*0*. 3 sig. figs. 119 to 121
- 12.0*0* 4 sig. figs. 11.99 to 12.01

Tro: Chemistry: A Molecular Approach, 2/e

Practice -- Counting Significant Figures
(Please check your answers on the next slide)

How many significant figures are in each number?

0.0035

1.080

2371

100.00

100,000

100,000.

Practice -- Counting Significant Figures

How many significant figures are in each number?

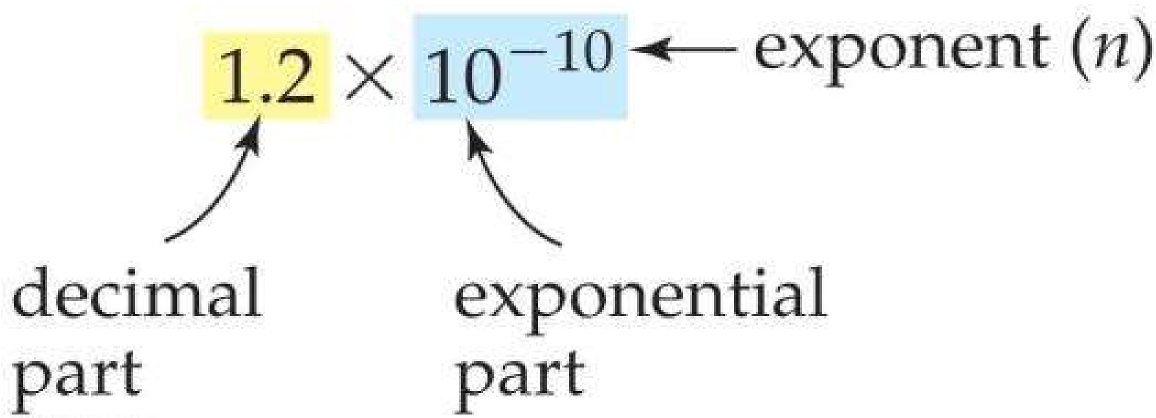
0.0035	two
1.080	four
2371	four
100.00	five
100,000	one
100,000.	six

When you have:

Leading zeros, ambiguous trailing zeros,
or just too many zeros ….

A better way to write a number: Scientific Notation

- A **decimal part**:
  a number that is between 1 and 10 (but less than 10).
  - Shows only the significant figures
- An **exponential part**:
  10 raised to an integer power
  - Shows only the size ("order of magnitude") of the number



The anatomy of 10^n

A positive exponent *n* means:
1 <u>multiplied</u> by **10** *n* times

$$10^0 = 1$$

$$10^1 = 1 \times 10 = 10$$

$$10^2 = 1 \times 10 \times 10 = 100$$

$$10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

A negative exponent *n* means:
1 <u>divided</u> by **10** *n* times

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$

$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$

Decimal Format to Scientific notation

0.000001234 ---> 1.234×10^{-6}

To get a number with only one nonzero digit before decimal point

- The decimal point in 0.000001234 had to move to the **right** by 6 positions
- Or, there are 6 zeros (including the one before the decimal point) before the first nonzero digit
- So the exponent is –6

Implied decimal point

1234000 ----> 1.234×10^{6}

To get a number with only one nonzero digit before decimal point

- The (implied) decimal point in 1234000 had to move to the **left** by 6 positions
- So the exponent is +6 (i.e. just 6)

Decimal Format to Scientific Notation -- Another couple of examples

$$5983 = 5.983 \times 10^{3}$$

3 2 1

$$0.00034 = 3.4 \times 10^{-4}$$

1 2 3 4

Again:

- If the decimal point had to be moved to the **left**, the exponent is **positive**.
- If the decimal point had to be moved to the **right**, the exponent is **negative**.

Calculations with exponents

For any number (10 here) raised to various powers

- **Multiplication**: add powers

  $10^7 \times 10^8 = 10^{7+8} = 10^{15}$

- **Division**: subtract powers

  $10^7 / 10^8 = 10^{7-8} = 10^{-1}$

- Power to another power (**exponentiation**): multiply powers

  $(2 \times 10^4)^3 = 2^3 \times (10^4)^3 = 8 \times 10^{4 \times 3} = 8 \times 10^{12}$

  Both terms are raised to the power that applies to the parentheses

Practice
(Please check your answers on the next slide)

Convert to scientific notation.

Remember: You should only include the significant digits in the decimal part

350078000

350078000.

0.00043210

22.6×10^{-3}

0.00856×10^{7}

Practice
(Please check your answers on the next slide)

Convert to scientific notation.

Remember: You should ***only*** *include the significant digits in the decimal part*

350078000 $3.50078x10^{8}$

350078000. $3.50078000x10^{8}$

0.00043210 $4.3210x10^{-4}$

22.6×10^{-3} $2.26x10^{-2}$

0.00856×10^{7} $8.56x10^{4}$

Practice (Please check your answers on the next slide)

Convert to Standard (i.e. "decimal") notation.

Remember: You need to keep all the significant digits in the decimal part

5.19×10^{3}

5.190×10^{3}

5.1900×10^{3}

7.473×10^{-2}

0.7473×10^{-2}

Practice

Convert to Standard (i.e. “decimal”) notation.

Remember: You need to keep all the significant digits in the decimal part

5.19×10^3 5190

5.190×10^3 5190.

5.1900×10^3 5190.0

7.473×10^{-2} 0.07473

0.7473×10^{-2} 0.007473

Exact numbers

- They have an unlimited (infinite) number of significant figures
- For “Exact Numbers”, we don’t need to identify the significant digits
- All listed digits are significant, and ….
- We can assume an infinite number of implied, significant zeros wherever needed

If 2.54 is supposed to be exact, then it’s like

2.540000000000……….

What numbers are "exact"?

- Exact counts (not estimates of a count) of discrete objects
  - 7 pencils → 7.00000000……..
- Integral numbers (integers) that are part of an equation
  - The "2" in y = 2x → 2.000000000……..
- Defined quantities
  - Speed of light is now actually defined and fixed: 299,792,458 meters/second
  - It has infinite number of significant figures, not just 9
  - Can be used like 299792458.0000000000000000……. meters/second
- *Some conversion factors are defined quantities, while others are not.*
  - For example, 1 in. = 2.54 cm exactly
  - It has infinite number of significant figures, not just 3
  - Can be used like 1 in. = 2.54000000000000000000……. cm

By the way, the "1" in conversion relationships is a "count", So it is exact

But be careful with exact numbers

- What if I am given a defined quantity with less digits than it was defined with?
- Such as, if I am given the speed of light as 3.00×10^8 meters/second?
  - Then we treat it as a normal, not-exact number

- 1 pound (lb) is defined to be exactly 0.45359237 kilograms
- If we express the relationship to show how many lb there are in 1 kg:

  1 kg = 2.2046226218487758072297380134503…….. lb !!!!!!!!

  (it's the reciprocal of 0.45359237, i.e. 1/0.45359237, and goes on forever)
  - In this form, there is no exact conversion factor.
- So if we are given, instead, 1 kg = 2.2046 lb, the number "2.2046" has only 5 significant figures, not infinite

Practice (Please check your answers on the next slide)

Estimate the number of sig. figs:

42.301 ounces

0.000960 seconds

8.91500×10^3 meters

103052.0 gallons

28 students

Value of a dollar bill

0.75 mile

Practice

Estimate the number of sig. figs:

42.301 ounces	5
0.000960 seconds	3
8.91500×10^3 meters	6
103052.0 gallons	7
28 students	Infinite
Value of a dollar bill	Infinite
0.75 mile	2

Significant Figures in Calculations

Rules for Rounding:

- When numbers are used in a calculation, the result is rounded to reflect the significant figures of the *data*.

  -- We can of course be asked or required to round off to some other level of precision

  -- But normally we must get rid of all (and only) non-significant figures

- For calculations involving multiple steps, round only the final answer—*do not round off between steps*.

  This practice prevents small rounding errors from affecting the final answer.

  - This is why sometimes the last digit of the "correct answer" on a test may be off a little

Rules for Rounding:

- Round down if the first digit dropped is 4 or less
- Round up if the first digit dropped is 6 or more
- Round up if the first digit to be dropped is 5, with at least one nonzero digit anywhere to its right
- If the first digit to be dropped is 5 and any digits to its right are zeros

  -- we are used to always rounding it up

  -- it's ok for an introductory course

  -- But it's more complicated than that.

  -- We can do the wrong thing at this level, but know that it's mathematically not right to always round up in this case

  -- By Convention we must round to the nearest even integer

Practice
(Please check your answers on the next slide)

Round off the following numbers

1.237651 to 4 sig. figs

0.77555 to 3 sig. figs.

13.4219 to 2 sig. figs

114.1 to 2 sig figs

Practice

Round off the following numbers

1.237651 to 4 sig. figs 1.238

0.77555 to 3 sig. figs. 0.776

13.4219 to 2 sig. figs 13

114.1 to 2 sig figs 110

How many sig figs in the result of a calculation?

General rule: ***The lousiest one wins!***

- "lousiest" = least precise
- "least precise" means different things for multiplication & division versus addition & subtraction

  - **For multiplication & division**
    - **Precision = number of sig figs**

      12.3 less precise than 123.4

  - **For addition & subtraction**
    - **Precision = the last (rightmost) significant decimal place**

      123.4 is less precise than 3.45

      even though 123.4 has more sig figs

How many sig figs in the result of a calculation?

For Multiplication and Division:

The result of multiplication or division carries the same number of significant figures as the factor (number) with the fewest significant figures.

Significant Figures in Multiplication & Division

"The result of multiplication or division carries the same number of significant figures as the factor (number) with the fewest significant figures"

$$5.02 \times 89.665 \times 0.10 = 45.0118 = 45$$

(3 sig. figures) (5 sig. figures) (2 sig. figures) (2 sig. figures)

In this particular example:

The result (in blue) is rounded to two significant figures to reflect the least precisely known factor (0.10), which has two significant figures.

Significant Figures in Multiplication & Division

"The result of multiplication or division carries the same number of significant figures as the factor (number) with the fewest significant figures"

$$5.892 \div 6.10 = 0.96590 = 0.966$$

(4 sig. figures) (3 sig. figures) (3 sig. figures)

In this particular example:

The result (in blue) is rounded to three significant figures to reflect the least precisely known factor (6.10), which has three significant figures.

Practice (Please check your answers on the next slide)

Report the result of the following operations in the correct number of sig. figs

23.6×2.789

0.77555×2.89

$3.22 \times 10^{-2} \times 9.8$

Practice

Report the result of the following operations in the correct number of sig. figs

23.6×2.789 65.8

0.77555×2.89 2.24

$3.22 \times 10^{-2} \times 9.8$ 0.32

Significant Figures in Addition & Subtraction

For Addition and Subtraction:

The last decimal place of the result corresponds to that of the number whose last significant decimal place has the highest value (e.g. 1/10 is higher than 1/100).

"For addition and subtraction, the last decimal place of the result corresponds to that of the number whose last significant decimal place is the highest"

No significant digit after this

5.74 — Last significant digit is at the hundredths (1/100) place

0.823 — Last significant digit is at the thousandths (1/1000) place

$+\,2.651$ — Last significant digit is at the thousandths (1/1000) place

$9.214 = 9.21$ — Last significant digit must be at the hundredths (1/100) place

No significant digit after this

We round the intermediate answer (in blue) to two decimal places after the point because 5.74 is the quantity whose last significant decimal is the largest, and ends at the second decimal place after the decimal point.

"For addition and subtraction, the last decimal place of the result corresponds to that of the number whose last significant decimal place is the highest"

$$\begin{array}{r} 4.8 \\ -\,3.965 \\ \hline 0.835 \end{array} = 0.8$$

We round the intermediate answer (in blue) to one decimal place after the point because 4.8 is the quantity whose last significant decimal is the largest, and ends at the first decimal place after the decimal point.

You might see a different rule about addition & subtraction:

"The result has as many decimal places as the quantity with the fewest decimal places"

But that applies only if:

- all the quantities have explicit decimal points!
- and "fewest decimal places" refers to after the decimal point

So that's basically wrong.

Example:

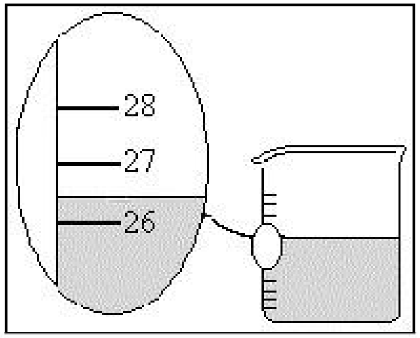
Last significant decimal place is the 10s (tens) place (Lousier one wins)

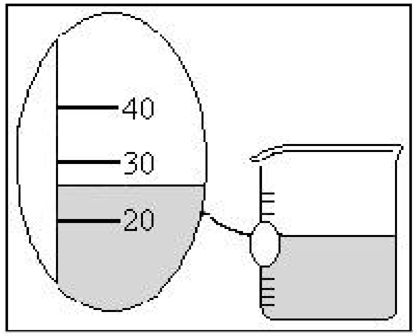
130 + 12 = 142 → 140

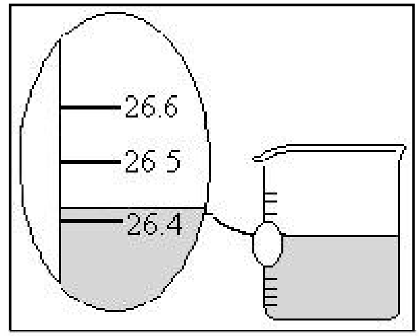
Last significant decimal place is the 10s (tens) place

Last significant decimal place is the 1s (ones) place

The beakers below have different precisions.







All numbers are in "milliliters" (mL)

You pour the water from these three beakers into one container. What is the volume in this container reported to the correct number of significant figures?

a) 78.817 mL
b) 78.82 mL
c) 78.8 mL
d) 79 mL

Multiplication/Division & Addition/Subtraction Combined

In calculations involving both multiplication/division and addition/subtraction,

1. Do the steps in parentheses first;

    "parentheses" may not be explicit;
    operations in the numerator or the denominator are same as parentheses

2. If a number is sandwiched between a multiplication (or division) sign and an addition (or subtraction) sign, do the multiplication (or division) first

3. Determine the correct number of significant figures in the intermediate answer <u>without rounding</u>; then do the remaining steps.

    -- Put a bar under the last (and least) significant digit to keep track

In the calculation $3.489 \times (5.67 - 2.3)$

- Do the step in parentheses first: $5.67 - 2.3 = 3.\underline{3}7$
- Use the subtraction rule to determine that the intermediate answer has only one significant digit after the decimal point
- To avoid small errors, it is best not to round at this point; instead, underline the least (last) significant figure as a reminder.

$$3.489 \times 3.\underline{3}7 = 1\underline{1}.758 = 12$$

- Use the multiplication rule to determine that the intermediate answer (11.758) rounds to two significant figures (12) because it is limited by the two significant figures in $3.\underline{3}7$.

Multiplication/Division & Addition/Subtraction Combined

$$2.01 \times (9.99 - 9.98) + 0.123$$

$$9.99 - 9.98 = 0.01$$

$$2.01 \times 0.01 = 0.0\underline{2}01$$

$$0.0\underline{2}01 + 0.123 = 0.1\underline{4}31 \rightarrow 0.14$$

Practice Perform the Following Calculations to the Correct Number of Significant Figures

Check your answer on the next slide

a) $1.10 \times 0.5120 \times 4.0015 \div 3.4555$

b)
$$\begin{array}{r} 0.355 \\ +105.1 \\ -100.5820 \\ \hline \end{array}$$

c) $4.562 \times 3.99870 \div (452.6755 - 452.33)$

d) $(14.84 \times 0.55) - 8.02$

Practice Perform the Following Calculations to the Correct Number of Significant Figures

a) $1.10 \times 0.5120 \times 4.0015 \div 3.4555 = 0.65219 = 0.652$

b)
$$\begin{array}{l} \quad 0.355 \\ +105.1 \\ -100.5820 \\ \hline 4.8730 = 4.9 \end{array}$$

c) $4.562 \times 3.99870 \div (452.6755 - 452.33) = 52.79904 = 53$

d) $(14.84 \times 0.55) - 8.02 = 0.142 = 0.1$

Using Dimensional Analysis to Convert Between Units

What is Dimensional Analysis?

Using units as a guide to solving problems

-- We will use it for unit conversions first

-- It will be extremely handy to solve quantitative problems, once you learn to see relationships as "conversion factors", and expand your idea of what could be a "unit"

- Units are multiplied, divided, and canceled like any other algebraic quantities.
- Always write every number with its associated unit.
- Always include units in your calculations, dividing them and multiplying them as if they were algebraic quantities.
- Do not let units appear or disappear in calculations. Units must flow logically from beginning to end.

For most conversion problems, we are given a quantity in some units and asked to convert the quantity to another unit. These calculations take the form:

$$\text{information given} \times \text{conversion factor(s)} = \text{information sought}$$

$$\cancel{\text{given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

Converting Between Units

- A "conversion factor" is a ratio of two quantities known to be equivalent.
  - For example 1 in. = 2.54 cm
- We construct the conversion factor by dividing both sides of the equality by one of the equivalent quantities and canceling the units.

$$2.54\ \text{cm} = 1\ \text{in.}$$

$$\frac{2.54\ \text{cm}}{1\ \text{in.}} = \frac{1\ \text{in.}}{1\ \text{in.}}$$

$$\frac{2.54\ \text{cm}}{1\ \text{in.}} = 1$$

Or we could construct:

$$\frac{1\ \text{in.}}{2.54\ \text{cm}} = 1$$

A conversion factor (ratio) itself is equal to unitless 1

So multiplying a quantity with it doesn't change the quantity

- Make sure the unit you want convert from (i.e. “get rid of”) is in the denominator (bottom part) of the conversion factor, so it cancels the starting unit and

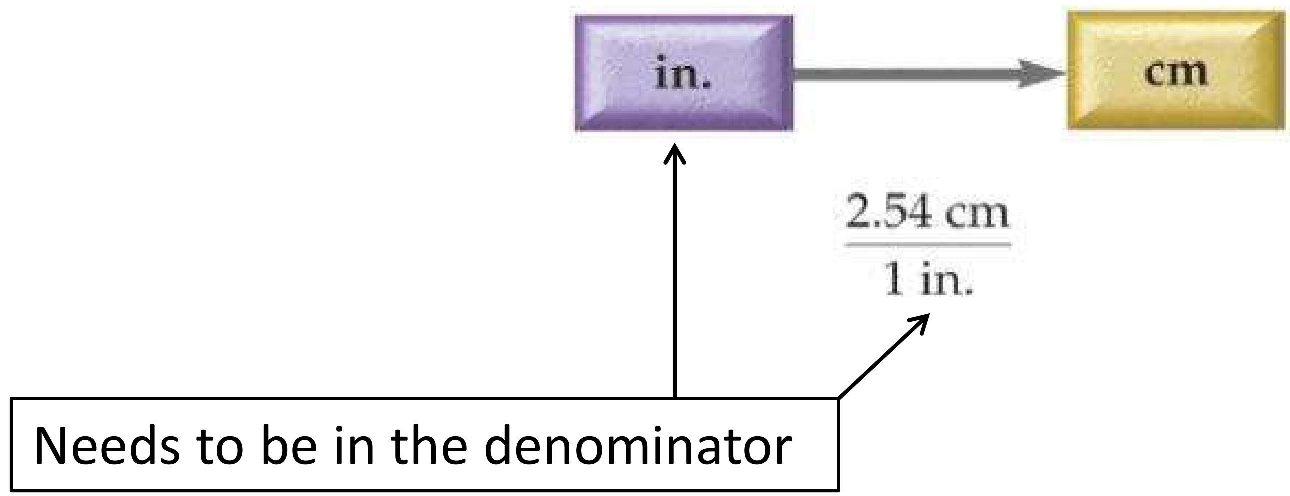
- The unit you want to convert to is in the numerator (top part) of the conversion factor

Convert to

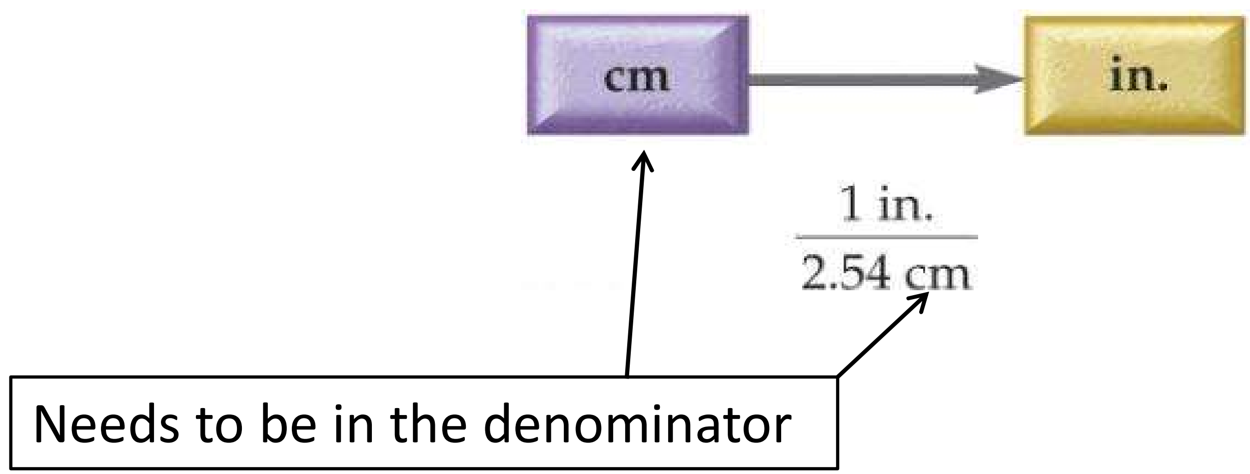
$$44.7\ \cancel{\text{cm}} \times \frac{1\ \text{in.}}{2.54\ \cancel{\text{cm}}} = 17.6\ \text{in.}$$

Convert from

- To convert from inches to centimeters:

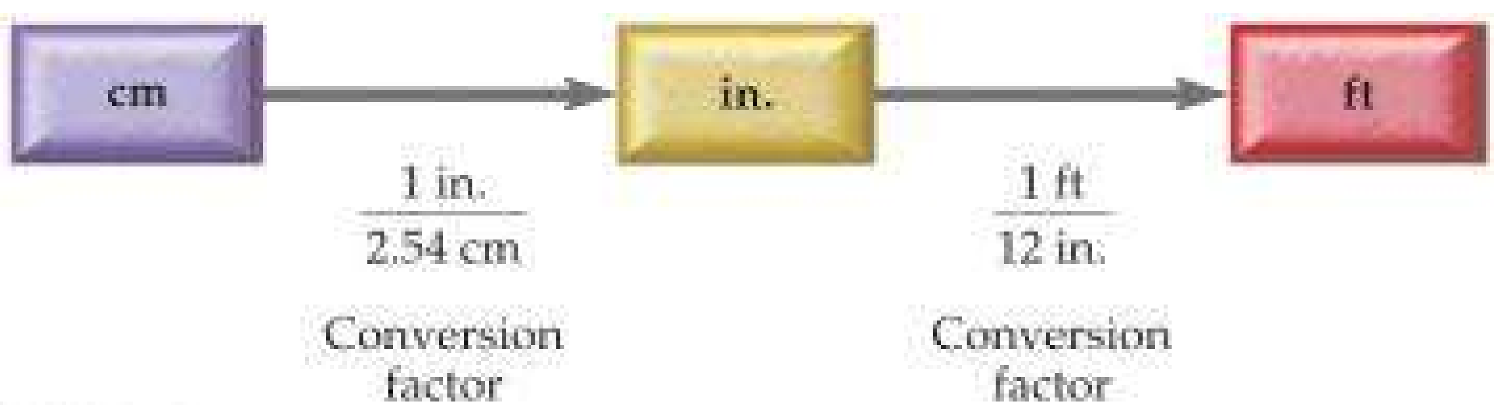


- To convert from centimeters to inches:



Solving Multistep Unit Conversion Problems

- Each step in the solution map should have a conversion factor with the units of the previous step in the denominator and the units of the following step in the numerator.
- If we are given the conversion between inch and cm, and inch and ft, but **not** between cm and ft, we construct the following solution map



$$194\ \text{cm} \times \frac{1\ \text{in.}}{2.54\ \text{cm}} \times \frac{1\ \text{ft}}{12\ \text{in.}} = 6.36\ \text{ft}$$

The Basic Units of Measurement

The unit system for science measurements, based on the metric system, is called the International System of Units (*Système International d'Unités*) or ***SI* units**.

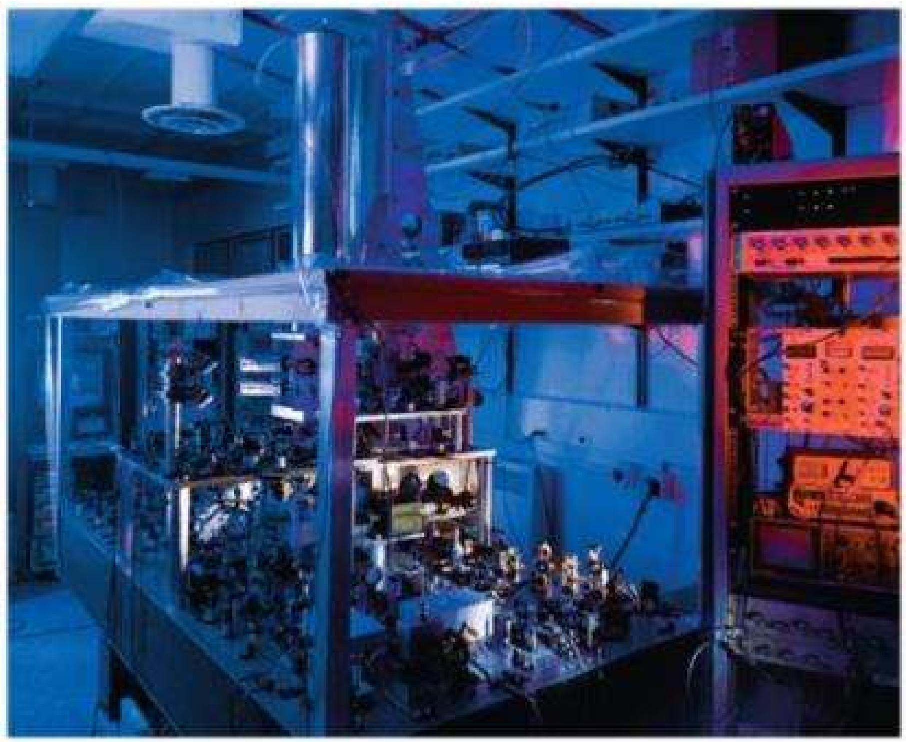
More relevant for introductory chemistry → Length, Mass, Time, Temperature, Amount of substance

The SI base units		
Property	**Unit**	**Symbol**
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

[You are not expected to know or memorize the details on this slide. Just the ideas.]

The standard of time

- The second is defined, using an atomic clock, as the duration of 9,192,631,770 periods of the radiation emitted from a certain transition in a cesium-133 atom.
- No longer "one-86400th of a day"



[You are not expected to know or memorize the details on this slide. Just the ideas.]

The standard of length

- The definition of a **meter**, established by international agreement in 1983, is the distance that light travels in vacuum in 1/299,792,458 second.
- Because:
  The speed of light in vacuum is now **defined** to be 299,792,458 m/s)
- It also relies what a “second” is exactly
  - That’s another standard
- There is no longer a “standard meter” kept in a vault

[You are not expected to know or memorize the details on this slide. Just the ideas.]

The standard of mass

- As of May 20, 2019, the definition of the **kilogram** is based on a fixed assignment of the Planck constant as $6.62607015 \times 10^{-34}$ $kg \cdot m^2 \cdot s^{-1}$ (a fundamental constant of the universe), yielding a definition of the kilogram in terms of the second and the meter in a consistent system of units based on physical constants only.
- There is no longer a “standard kilogram” kept in a vault

The standard of mass The kilogram is defined as the mass of a block of metal kept at the International Bureau of Weights and Measures at Sèvres, France. A duplicate is kept at the National Institute of Standards and Technology near Washington, D.C.

By the way, weight is not the same as mass

- The kilogram is a measure of mass, which is different from weight.
- The **mass** of an object is a measure of the quantity of **matter** within it.
- The weight of an object is a measure of the gravitational pull on that matter.
- Consequently, **weight** depends on **gravity** while mass does not.

Remember? SI units for any quantity are related by powers of 10.
Well, those powers of 10 have names and symbols.

SI Prefix Multipliers

Need to be memorized

Prefix	Symbol	Meaning	Multiplier	
tera-	T	trillion	1,000,000,000,000	(10^{12})
giga-	G	billion	1,000,000,000	(10^{9})
mega-	M	million	1,000,000	(10^{6})
kilo-	k	thousand	1,000	(10^{3})
hecto-	h	hundred	100	10^{2}
deca-	da	ten	10	10^{1}
deci-	d	tenth	0.1	(10^{-1})
centi-	c	hundredth	0.01	(10^{-2})
milli-	m	thousandth	0.001	(10^{-3})
micro-	μ	millionth	0.000001	(10^{-6})
nano-	n	billionth	0.000000001	(10^{-9})
pico-	p	trillionth	0.000000000001	(10^{-12})
femto-	f	quadrillionth	0.000000000000001	(10^{-15})

How do we use the SI prefixes to create units of appropriate size?

For example:
"nano" (symbol: n) means 10^{-9} (one billionth)
To make a unit that's a billionth of a meter (symbol: m), we define:
A "nanometer" (symbol: nm)
So we have:
1 nm = 10^{-9} m

Choosing Prefix Multipliers

- Scientists choose the prefix multiplier that is most convenient for a particular measurement.
- They pick a unit similar in size to (or smaller than) the quantity

A short chemical bond is about 1.2×10^{-10} m. Which prefix multiplier should you use? pico = 10^{-12}; nano = 10^{-9}

The most convenient one is probably the picometer (pm)

$$1.2 \times 10^{-10}\text{m} \times \frac{1\ \text{pm}}{10^{-12}\text{m}} = 120\ \text{pm}$$

Nanometer (nm) is a little too large a unit for the job (but not bad):

$$1.2 \times 10^{-10}\text{m} \times \frac{1\ \text{nm}}{10^{-9}\text{m}} = 0.12\ \text{nm}$$

Most reliable way to construct prefixed SI unit conversions

Simply use the numerical equivalent of the prefix

- You must get used to dealing with powers of 10
  - Negative power or not (10^9 or 10^{-12})
  - In the numerator or the denominator
  - You can't put off getting competent in that; used everywhere
- Put a "**1**" in front of the prefixed unit
  - The prefixed unit already has the prefix; it's fine already
- Put the numerical equivalent of the prefix (e.g. 10^{-3} for milli) in front of the <u>un-prefixed</u> (base) unit

Convert 2.5 μm to meters:

μ = "micro" = 10^{-6}

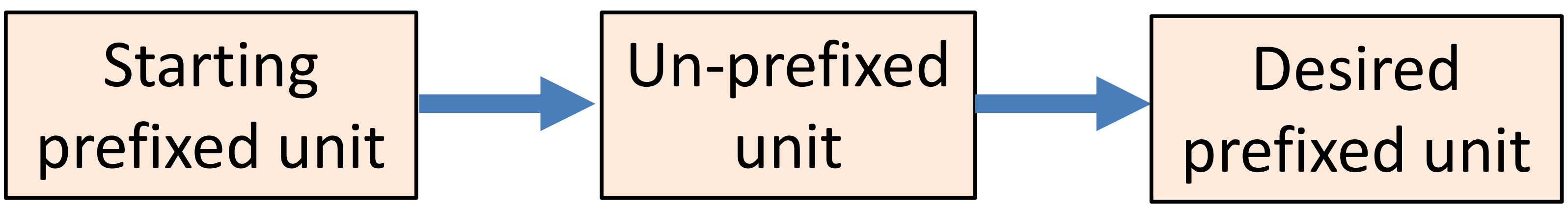
un-prefixed unit

$$2.5\ \cancel{\mu\text{m}} \times \frac{10^{-6}\ \text{m}}{1\ \cancel{\mu\text{m}}} = 2.5 \times 10^{-6}\ \text{m}$$

How to convert from one prefixed SI unit to another

Use the un-prefixed SI unit (meter, gram, second) as the intermediate unit

- First convert from the starting prefixed unit (e.g. km, pg, fs) to the un-prefixed SI unit (meter, gram, second)
- Then convert from un-prefixed SI unit (meter, gram, second) to the desired prefixed unit



nanometer — meter — picometer

$$0.121\ \text{nm} \times \frac{10^{-9}\ \text{m}}{1\ \text{nm}} \times \frac{1\ \text{pm}}{10^{-12}\ \text{m}} = 121\ \text{pm}$$

Practice

Check your answer on the next slide

Convert 1.76 yd to centimeters (1 m = 1.094 yd)

Practice: Convert 1.76 yd to centimeters (1 m = 1.094 yd)

• Sort the information	**Given:** **Find:**	1.76 yd length, cm
• Strategize	**Conceptual Plan:** **Relationships:**	yd → m → cm 1 m = 1.094 yd 1 cm = 10^{-2} m
• Follow the conceptual plan to solve the problem	**Solution:**	$1.79\ \text{yd} \times \dfrac{1\ \text{m}}{1.094\ \text{yd}} \times \dfrac{1\ \text{cm}}{10^{-2}\ \text{m}} = 160.8775\ \text{cm}$
• Sig. figs. and round	**Round:**	160.8775 cm = 161 cm
• Check	**Check:**	units are correct; number makes sense: cm << yd

Practice (Please check your answers on the next slide)

- How many mg does a 433 kg sample contain?
- How many km does a 1.25×10^8 cm contain?
- If someone's mass is 77.1 kg what is this mass in pounds (1 kg = 2.20 lb)?
- A jogger runs at an average speed of 6.50 miles/hour. Convert the speed to meters/second (1 km = 0.6215 mi)

Practice

- How many mg does a 433 kg sample contain?

$(433kg)x(10^3 g/1kg)x(1mg/10^{-3} g)= 4.33x10^8 mg$

- How many km does a 1.25×10^8 cm contain?

$(1.25 \times 10^8$ cm$)x (10^{-2}m /1cm)x(1km/10^3 m)= 1250km$

- If someone's mass is 77.1 kg what is this mass in pounds (1 kg = 2.20 lb)?

77.1kg x(2.20lb/1kg)= 169lb

- A jogger runs at an average speed of 6.50 miles/hour. Convert the speed to meters/second (1 km = 0.6215 mi)

$(6.50mi/hr)x(1km/0.6215mi)x(10^3 m/1km)x(1hr/3600s)=2.90m/s$

Volume and Volume Units

- Measures how much space is taken up by a sample or an object
- An important quantity in general
- Especially useful in chemistry
- Allows easy measurement of quantity, especially of liquids
- A popular unit is “Liters” (L)
- The SI prefixes are also used with liters

Practice

Convert 30.0 mL to quarts

(1 L = 1.057 qt)

Check your anwsers on the next slide

Tro: Chemistry: A Molecular Approach, 2/e

Practice – Convert 30.0 mL to quarts

• Sort information	**Given:** **Find:**	30.0 mL volume, qts
• Strategize	**Conceptual Plan:** **Relationships:**	mL → L → qt 1 L = 1.057 qt 0.001 L = 1 mL
• Follow the conceptual plan to solve the problem	**Solution:**	$30.0\ \text{mL} \times \frac{0.001\ \text{L}}{1\ \text{mL}} \times \frac{1.057\ \text{qt}}{1\ \text{L}} = 0.03171\ \text{qt}$
• Sig. figs. and round	**Round:**	0.03171 qt = 0.0317 qt
• Check	**Check:**	units are correct; and number makes sense: mL << qt

Practice (Please check your answers on the next slide)

- The volume of a certain raindrop is 0.0036 cL. Convert to microliters.
- A bottle contains 750 mL of soda. How many quarts of soda is that?
  (1 quart = 0.946353 L)

Practice

- The volume of a certain raindrop is 0.0036 cL. Convert to microliters.

$0.0036cL x (10^{-2}L/1cL) x (1\mu L/10^{-6}L) = 36\mu L$

- A bottle contains 750 mL of soda. How many quarts of soda is that?
  (1 quart = 0.946353 L)

$750mL x (10^{-3}L/1mL) x (1quart/0.946353L) = 0.79quart$

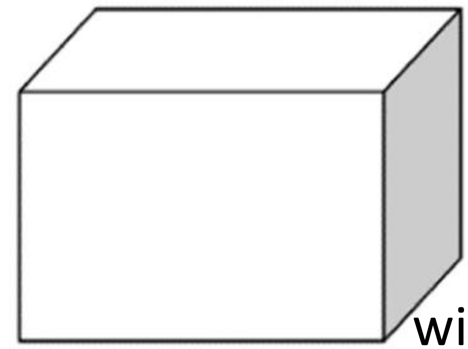
Volume as a Derived Unit

- A derived unit is formed from other units.
- Many units of **volume,** a measure of space, are derived from **length**.
- Any unit of length, when **cubed** (raised to the third power), becomes a unit of volume.

- How do we convert between different units of volume?
- First, let's look at obtaining conversion factors when the <u>units are raised to a power</u>
  - Such as with volume or area units derived from length

"height" *"width"*

Volume = length x length x length



Volume as a Derived Unit

- Cubic meters (m^3), cubic centimeters (cm^3), and cubic millimeters (mm^3) are all SI units of volume
- Liter is also an SI unit because

$$1\ L = 1\ dm^3$$

- Another useful unit to know is $1cm^3=1mL$

- How do we convert between different units of volume?
- First, let's look at obtaining conversion factors when the <u>units are raised to a power</u>
  - Such as with volume or area units derived from length

Conversion with Units Raised to a Power -- for Volume

- Volume has the units of length-cubed.
- So we cube both sides of the length conversion equation to obtain the proper conversion factor for volume.

$$(2.54\ \text{cm})^3 = (1\ \text{in.})^3$$

$$(2.54)^3\ \text{cm}^3 = 1^3\ \text{in.}^3$$

$$16.387\ \text{cm}^3 = 1\ \text{in.}^3$$

We can do the same thing in fractional form:

$$1255\ \text{cm}^3 \times \frac{(1\ \text{in.})^3}{(2.54\ \text{cm})^3}$$

↓

$$1255\ \text{cm}^3 \times \frac{1\ \text{in.}^3}{16.387\ \text{cm}^3} = 76.5851\ \text{in.}^3 = 76.59\ \text{in.}^3$$

Conversion with Units Raised to a Power -- for Area

- Area has the units of length-squared.
- So we square (instead of cube) both sides to obtain the proper conversion factor for area.
- The principle is the same as that for volume conversion factors

$$1 \text{ in} = 2.54 \text{ cm}$$

$$\downarrow$$

$$(1 \text{ in})^2 = (2.54 \text{ cm})^2$$

$$\downarrow$$

$$1^2 \text{ in}^2 = 6.4516 \text{ cm}^2$$

$$\downarrow$$

$$1 \text{ in}^2 = 6.4516 \text{ cm}^2$$

$$308 \cancel{\text{cm}^2} \times \frac{1 \text{ in}^2}{6.4516 \cancel{\text{cm}^2}} = 47.7 \text{ in}^2$$

How to obtain volume conversion factors (recap)

- Start from the length conversion relationship between the two units
- **Cube** both sides of the equation
- Any numbers in front of length units also get cubed

$$1\ \text{m} = 3.28084\ \text{ft}$$

↓

$$(1\ \text{m})^3 = (3.28084\ \text{ft})^3$$

↓

$$1^3\ \text{m}^3 = 3.28084^3\ \text{ft}^3$$

↓

$$1\ \text{m}^3 = 35.3147\ \text{ft}^3$$

How to obtain volume conversion factors -- important example, with SI prefixes

$1\ \text{cm} = 10^{-2}\ \text{m}$

↓

$(1\ \text{cm})^3 = (10^{-2}\ \text{m})^3$

↓

$1^3\ \text{cm}^3 = (10^{-2})^3\ \text{m}^3$

↓

$1\ \text{cm}^3 = 10^{-6}\ \text{m}^3$

So, when we see

$1\ cm^3$

Is it 1 centi(m^3) (one 100th of a m^3)? **No**

Or is it 1 (centimeter)^3 ? **Yes**

- “Centi” doesn’t simply mean 10^{-2} here
- It leads to 10^{-6} in the case of volume as we just saw
- And we learned how to handle it

Practice: Convert 5.70 L to cubic inches (1in = 2.54cm)

Check your solution on the next slide

Practice: Convert 5.70 L to cubic inches

• Sort information	**Given:** **Find:**	5.70 L volume, in.^{3}
• Strategize	**Conceptual Plan:** **Relationships:**	L → mL → cm^3 → in^3 1 mL = 1 cm^3, 1 mL = 10^{-3} L 1 cm = 2.54 in.
• Follow the conceptual plan to solve the problem	**Solution:**	$5.70\,\text{L} \times \dfrac{1\,\text{mL}}{10^{-3}\,\text{L}} \times \dfrac{1\,\text{cm}^3}{1\,\text{mL}} \times \dfrac{(1\,\text{in})^3}{(2.54\,\text{cm})^3} = 34\underline{7}.835\,\text{in}^3$
• Sig. figs. and round	**Round:**	$34\underline{7}.835$ in.^{3} = 348 in.^{3}
• Check	**Check:**	units are correct; number makes sense: in.^{3} << L

Practice

How many cubic centimeters are there in 2.11 yd^3 ?

1 yd = 36 in.

1 in. = 2.54 cm

Check your solution on the next slide

Practice – Convert 2.11 yd^3 to cubic centimeters

• Sort information	**Given:** **Find:**	2.11 yd^3 volume, cm^3
• Strategize	**Conceptual Plan:** **Relationships:**	$yd^3 \rightarrow in^3 \rightarrow cm^3$ 1 yd = 36 in. 1 in. = 2.54 cm
• Follow the conceptual plan to solve the problem	**Solution:**	$2.11\,yd^3 \times \frac{(36\,in)^3}{(1\,yd)^3} \times \frac{(2.54\,cm)^3}{(1\,in)^3}$ $= 16\underline{1}3210.75\,cm^3$
• Sig. figs. and round	**Round:**	$16\underline{1}3210.75\,cm^3$ $= 1.61 \times 10^6\,cm^3$
• Check	**Check:**	units and number make sense

Practice (please check your solution on the next page)

- If a room requires 25.4 square yards of carpeting, what is the area of the floor in units of ft^2 (square foot)? (3 ft = 1 yd)
- A sheet of notebook has an area of $603.22 cm^2$. What is the area of this paper in nm^2?

Practice

- If a room requires 25.4 square yards of carpeting, what is the area of the floor in units of ft^2?
  (3 ft = 1 yd)

$25.4yd^2 \; x(3ft/1yd)^2 = 229ft^2$

- A sheet of notebook has an area of $603.22cm^2$. What is the area of this paper in nm^2?

$(603.22cm^2)x(10^{-2}m/1cm)^2x(1nm/10^{-9}m)^2=6.0322x10^{16}nm^2$

General Problem-Solving Strategy

- **Sort.** Begin by sorting the information in the problem.
  - Identify the starting point (the given information).
  - Identify the end point (what you must find).
- **Strategize.** Create a series of steps to take you from the given information to what you are trying to find.
  - Use what is given as well as what you already know or can look up.
  - Could use a solution map, either explicitly or mentally

    Given → Solution Map → Find
- **Solve.** Carry out mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
- **Check.** Does this answer make physical sense?
  Are the units correct?

$1\ mL = 1\ cm^3$

This equivalence is an important one to remember

Let's see how we can verify it:

1 Liter (L) is equal to 1 dm^3

$$1\,mL \times \frac{10^{-3}\,L}{1\,mL} \times \frac{1\,dm^3}{1\,L} \times \frac{10^{-3}\,m^3}{1\,dm^3} \times \frac{1\,cm^3}{10^{-6}\,m^3} = 1\,cm^3$$

$$\left[\frac{10^{-1}m}{1\,dm}\right]^3 \qquad \left[\frac{1\,cm}{10^{-2}m}\right]^3$$

Density

- An important characteristic of materials
  - How much mass is "stuffed" into a given volume
  - So it's good to learn about it and be able to apply it
- But we will also use it to extend the use of dimensional analysis to solving quantitative problems

Density

The concept:
Mass of substance per unit volume of the substance.

Density is an important characteristic of materials

- We might desire high density if we need large mass in a small volume
  - a fluffy bullet or a cotton-ball paperweight won't be effective
- We might desire low density for a given amount of strength
  - A bike made of "carbon fiber" will be a lot lighter than one made of steel

Calculating Density

The density of a substance is the ratio of its mass to its volume.

We calculate the density of a substance by dividing the mass of a given amount of the substance by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad d = \frac{m}{V}$$

Common units: **g/cm³** or **g/mL**

Can be used for solids or liquids (g/cm³)

Usually used for liquids (g/mL)

	Material	Density (g/cm^3)
	Outer space	10^{-24}
Least dense solid	Graphene Aerogel	0.00016
	Lightest Silica aerogel	0.001
	Air	0.0012
Least dense metal	Lithium	0.534
	Alcohol	0.78
	Water	1.00
	Aluminum	2.70
	Lead	11.34
Densest liquid	Mercury	13.5
Densest substance	Osmium	22.59
	Neutron star	10^{14}

Example

If a sample of liquid has a volume of 22.5 mL and a mass of 27.2 g, what is its density?

$$d = \frac{m}{V} = \frac{27.2\ \text{g}}{22.5\ \text{mL}} = 1.21\ \text{g/mL}$$

A Solution Map Involving the Equation for Density

The "solution map" just shows that, given mass (*m*) and volume (*V*), we use the equation *d*=*m*/*V* to find *d*

given: | use: | to find:

m, V → $d = \frac{m}{V}$ → d

or to find volume (*V*), given density (*d*) and mass (*m*)

m, d → $V = \frac{m}{d}$ → V

or to find mass(m), given density (d) and volume (V)

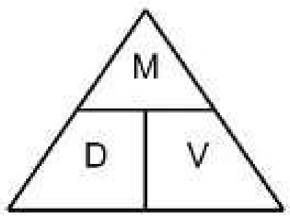
d, V → $m = d \cdot V$ → m

The equations

$$d = \frac{m}{V} \qquad V = \frac{m}{d} \qquad m = d \cdot V$$

The equations all represent the same relationship, just rearranged algebraically

Wanting to avoid “algebra” to avoid mistakes in favor of pre-college “magic shortcuts” like



is counterproductive.

- You will soon have to deal with basic algebra; be ready
- Those who resort to shortcuts end up often using them incorrectly anyway (I have seen it enough times in lab reports)

Density as a Conversion Factor

- When things are related by simple multiplication and division, i.e. by direct or inverse proportionality, we can use a ratio like density as a "conversion factor"
- Between mass (*m*) and volume (*V*) in this case

Example:

For a liquid substance with a density of 1.32 g/cm^3, what volume should be measured to deliver a mass of 68.4 g?

$$68.4\ \cancel{g} \times \frac{1\ cm^3}{1.32\ \cancel{g}} = 51.8\ cm^3$$

Note how we place g and cm^3 in order to cancel g and obtain cm^3

Example:

For a liquid substance with a density of 1.32 g/cm^3, what mass would be delivered by a volume of 51.8 cm^3?

$$51.8\ \cancel{cm^3} \times \frac{1.32\ g}{1\ \cancel{cm^3}} = 68.4\ g$$

Density as a Conversion Factor

Using density as a "conversion factor" is an example of using "dimensional analysis" to solve quantitative problems involving proportionality

- We will use dimensional analysis that way
- Not just with density problems

- Dimensional analysis is more efficient, general, and powerful than setting up proportions
  - Wean yourself off setting up proportions

- Much “safer”; helps you avoid mistakes
  - Especially when you are tired/sleep-deprived/panicked

- **Practice**: What is the mass in kg of 173,231 L of jet fuel whose density is 0.768 g/mL?

Check your solution on the next slide

Practice: What is the mass in kg of 173,231 L of jet fuel whose density is 0.768 g/mL?

• Sort information	**Given:** **Find:**	173,231 L density = 0.768 g/mL mass, kg
• Strategize	**Conceptual Plan:** **Relationships:**	L → mL → g → kg 1 mL = 0.768 g, 1 mL = 10^{-3} L 1 kg = 1000 g
• Follow the conceptual plan to solve the problem	**Solution:**	$173{,}231\,\text{L} \times \frac{1\,\text{mL}}{10^{-3}\,\text{L}} \times \frac{0.768\,\text{g}}{1\,\text{mL}} \times \frac{1\,\text{kg}}{1000\,\text{g}}$ $= 1.3\underline{3}041 \times 10^{5}\,\text{kg}$
• Sig. figs. and round	**Round:**	$1.3\underline{3}041 \times 10^{5} = 1.33 \times 10^{5}$ kg
• Check	**Check:**	units and number make sense

Practice – Calculate the Following (check your answer on the next slides

- How much does 3.0×10^2 mL of ether weigh? (d = 0.71 g/mL)

- What volume does 100.0 g of marble occupy? (d = 4.0 g/cm^3)

Practice - How much does 3.0×10^2 mL of ether weigh?

Sort information	**Given:** **Find:**	3.0×10^2 mL density = 0.71 g/mL mass, g
Strategize	**Conceptual Plan:** **Relationships:**	mL → g 1 mL = 0.71 g
Follow the conceptual plan to solve the problem	**Solution:**	$3.0 \times 10^2 \cancel{\text{mL}} \times \frac{0.71\text{ g}}{1 \cancel{\text{mL}}} = 2\underline{1}3\text{ g}$
Sig. figs. and round	**Round:**	2.1×10^2 g
Check	**Check:**	units are correct; number makes sense: if density < 1, mass < volume

Practice – What volume does 100.0 g of marble occupy?

Sort information	**Given:** **Find:**	m = 100.0 g density = 4.0 g/cm^3 volume, cm^3
Strategize	**Conceptual Plan:** **Relationships:**	g → cm^3 1 cm^3 = 4.0 g
Follow the conceptual plan to solve the problem	**Solution:**	$1.000 \times 10^2 \cancel{g} \times \frac{1\,cm^3}{4.0\,\cancel{g}} = 2\underline{5}\ cm^3$
Sig. figs. and round	**Round:**	25 cm^3
Check	**Check:**	units are correct; number makes sense: if density > 1, mass > volume

Practice
Please check your solution on next page

- Determine the density of an object that has a mass of 149.8 g and displaces 121.1 mL of water when placed in a graduated cylinder.

Practice

- Determine the density of an object that has a mass of 148.8 g and displaces 121.1 mL of water when placed in a graduated cylinder.

If an object displaces 121.1mL of water, it means that the object has 121.1mL (or 121.1cm^3).

(mL → cm^3 because it's more natural for solids)

$d=m/V = (148.8g)/121.1cm^3 = 1.229g/cm^3$